# *Optimal battery charge/discharge strategies for consumers and suppliers*

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# Major challenges

- To understand the interplay between the electricity market (price) and electricity generation, and what role storage might have in management of the power system.
- To understand the dynamics of smart grids consisting of a network of *prosumers* interacting in the physical, cyber and social layers

- Prosumer = Producer/Consumer

# Fundamental questions

- How can a market be effectively regulated, controlled/stabilised and incentivised?

- What is the effect of price on a power system?
  - Can price be used to regulate a complicated network containing a mix of generators, storers, prosumers?
  - Is price something we can impose on the system as an *exogenous* control variable or is it necessarily *endogenous*, a product of system dynamics?

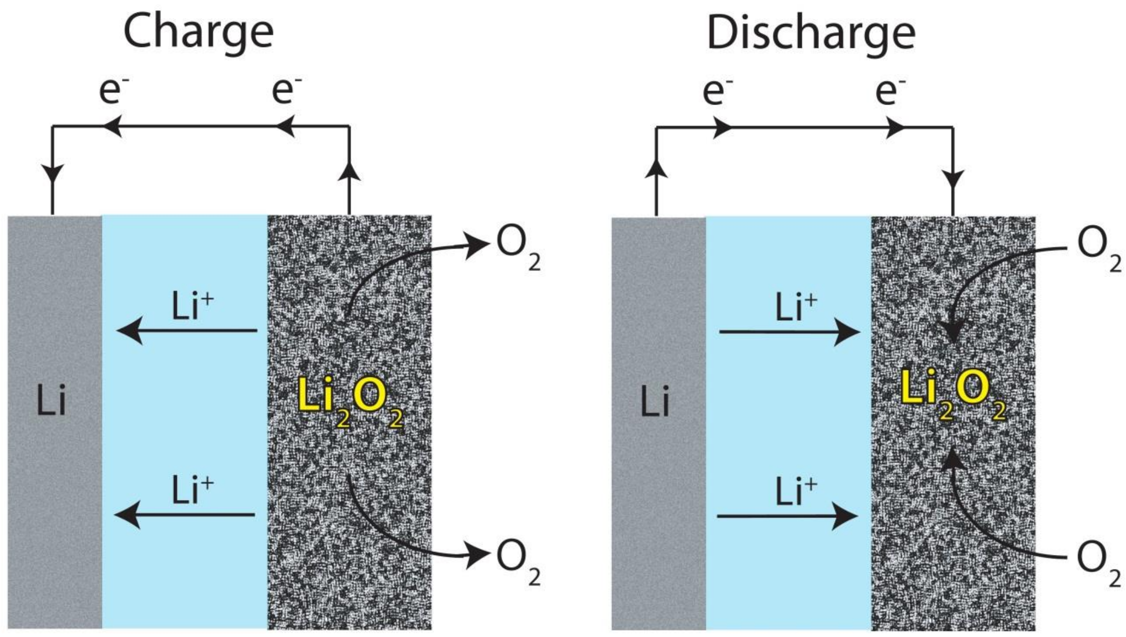
# Model (microeconomic) problem

- Given a prosumer with a storage battery (perhaps in an electric/hybrid vehicle) and an exogenous price $p(t)$ what is the optimal charging/discharging to minimise cost? How should the price be chosen to induce a given behaviour in the prosumer?

- In this talk we consider the charging problem and discuss the classical calculus of variations approach and its limitations

# Battery types

- Lead-acid
- Li Ion e.g. $LiCoO_2$
- NaS
- NiCd
- ZnBr
- $LiO_2$
- LiS




Schematic of lithium-air battery charge and discharge cycles

# Battery models

- Batteries are complicated beasts !
- Modelling approaches include
  - Electrochemical and thermal modelling of the electrolyte, electrodes and their interaction
  - Dynamical modelling – macro modelling of battery behaviour, often involving equivalent circuit diagrams
- Modelling of batteries is a well established and growing field.
- Little analytical work possible, simulation (often through Matlab and Maple) is main approach

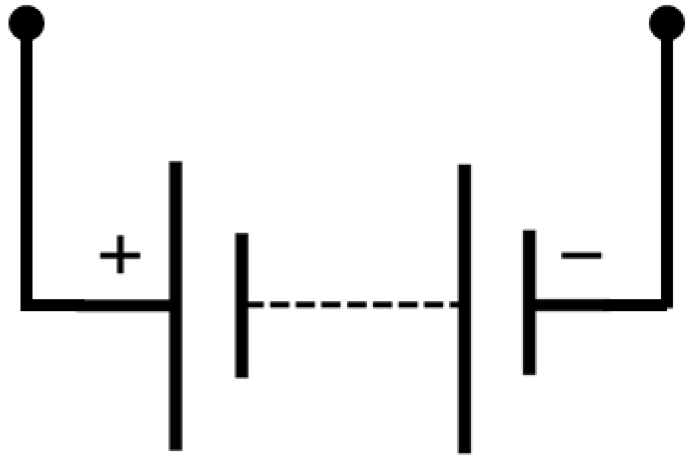
# Charging regimes

- Constant current
- Constant voltage
- High current decreased exponentially
- Constant current/constant voltage
- Constant current/constant voltage/constant current
- Pulsed charging
- Quick charging

- Each regime has characteristic charging time and effect on battery temperature and lifetime
- Each battery has its own attributes and manufacturer-recommended charging regimes

# Battery models

- State variable: $S$ = state of charge
- $S = 0$ battery fully discharged
- $S = 1$ battery fully charged
- (Ignore battery temperature $\theta$)
- Applied current and voltage $I(t), V(t)$
  - $I(t) > 0$ - charging
  - $I(t) < 0$ - discharging
  - $I(t)$, $V(t)$ are not independent
- Time interval $[t_s, t_e]$



# Single storage supplier fixed market

- Supplier *price taker*
- Prices: exogenous variables, set by power system operator
- Two prices in, say, £ per KWh  (i.e. money/energy):
  - $p_o(t)$ offer price i.e. price the storage supplier sells electricity
  - $p_b(t)$ bid price i.e. price the storage supplier buys electricity
  - In general $p_b(t) \geq p_o(t)$
- Forward pricing on $\mathrm{T} = [t_s, t_e]$
- Here we consider $p_o(t) = p_b(t)$, which we write $p(t)$

# Charging optimisation

- $S(t)$ state of charge, $0 \le S(t) \le 1,\ Q(t) = Q_{MAX}\, S(t)$

- $C = \int_{t_s}^{t_e} p(t)\, W(t) dt, \qquad W(t) = G(S(t), \dot{S}(t))$

- For a simple-battery
  - $W(t) = I(t)V(t)$
  - $V(t) = V_{OC} + R_b\, I(t)$
  - $I(t) = \ Q_{MAX}\, \dot{S}(t)$
  - $G = G(\dot{S}(t)) = \ (V_{OC} + R_b Q_{MAX}\, \dot{S}(t)) Q_{MAX}\, \dot{S}(t)$
  - $G$ convex for $R_b > 0$

# Classical Calculus of Variations Approach

- Theorem. Let $L(t, S, V)$ be a twice continuously differentiable function with respect to $(t, S, V)$ which is convex with respect to $(S, V)$. Then the functional $\int_{t_s}^{t_e} L\left(t, S(t), \dot{S}(t)\right) dt,\ \ S(t_s) = S_s,\ S(t_e) = S_e$ has a minimum path S$(t)$ that satisfies the Euler-Lagrange equation $\frac{d}{dt} L_V\left(t, S(t), \dot{S}(t)\right) = L_S(t, S(t), \dot{S}(t))$ with boundary conditions $S(t_s) = S_s,\ S(t_e) = S_e$

- Note: The Euler-Lagrange equation may be written as $\dot{Y}(t) = L_S(t, S(t), \dot{S}(t))$ , $Y(t) = L_V\left(t, S(t), \dot{S}(t)\right)$

# Q1: optimal charging

- For a given price $p(t)$, what is the optimal $S(t)$ (within a given class of approved charging regimes)? Is the charging regime unique, and, if not, can we characterize the degree of degeneracy?

- Euler-Lagrange equation

$$\frac{d}{dt}[p(t)G_{\dot{S}}] - p(t)G_S = 0, S(t_s) = S_s, S(t_e) = S_e$$

Note: $G_S = \partial G/\partial S$ etc

- For simple battery

$$S(t) = S_s + K \int_{t_s}^{t} \frac{ds}{p(s)} - \frac{V_{OC}}{2\,Q_{MAX}\,R_b}(\mathrm{t} - \mathrm{t_s}), \qquad \mathrm{S(t_e) = S_e}$$

# Q2: Optimal price

- For a given $S(t)$ what is the price $p(t)$ for which $S(\mathrm{t})$ is optimal? And is $p(t)$ unique and, if not, can we characterize the degree of degeneracy?

- $p(t) = p(t_s)\,\frac{G_{\dot{S}}(\mathrm{t}_s)}{G_{\dot{S}}(t)}\ \exp\left[\int_{t_s}^{t} \frac{G_S(s)}{G_{\dot{S}}(s)}\,ds\right]$

- For simple battery

$$p_{(t)} = p(t_s)\,\frac{V_{OC}+2\,Q_{MAX}\,R_b\,\dot{S}(t_s)}{V_{OC}+2\,Q_{MAX}\,R_b\,\dot{S}(t)}$$

# Q3: Specifying power $W(t)$

- For a given power $W(t)$, what is $S(t)$ (and is it unique and in a given class of approved charging functions)?

- $G(S, \dot{S}) = W(t)$ is an implicit differential equation for $S(t)$, which can be solved with one boundary condition $S(t_s) = S_s$. It's then possible in principle to determine the price function inducing this charging function providing $W(t)$ is compatible with $S(t_e) = S_e$.

- Simple battery:

- $S(t) = S_s + \frac{1}{2\, Q_{MAX} R_b} \int_{t_s}^{t} \left({V_{OC}}^2 + 4\, R_b W(s)\right)^{\frac{1}{2}} - V_{OC}\, ds$

# Q3 cont'd

- For a given power $W(t)$, what is the price $p(\mathrm{t})$ that induces $W(t)$?

- Apply the previous theory.

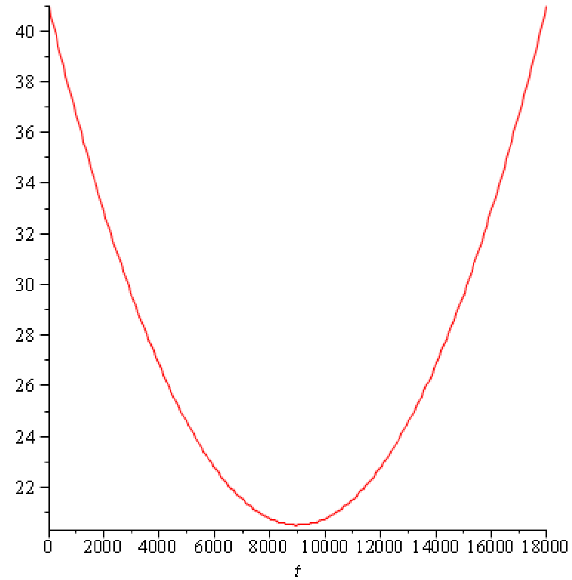
- For a simple battery:

$$p(t) = p(t_s)\left(\frac{V_{OC}^2+4\,R_b\,W(t_s)}{V_{OC}^2+4\,R_b\,W(t)}\right)^{\frac{1}{2}}$$

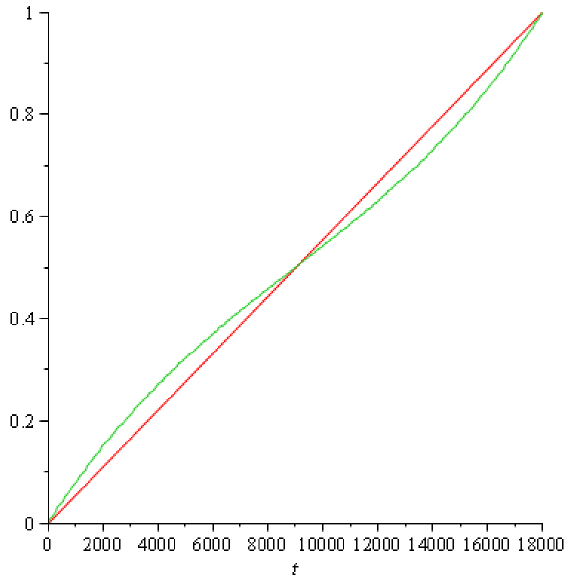
$$C = p(t_s)\ \int_{t_s}^{t_e}\left(\frac{V_{OC}^2+4\,R_b\,W(t_s)}{V_{OC}^2+4\,R_b\,W(s)}\right)^{\frac{1}{2}}\ W(s)ds$$

# Example

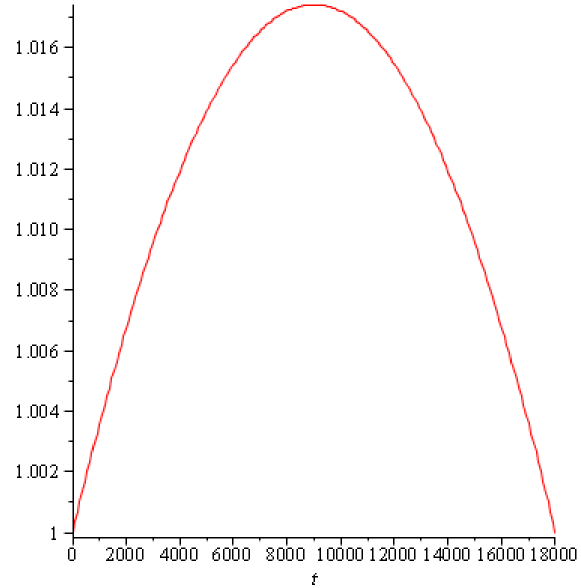
$W$



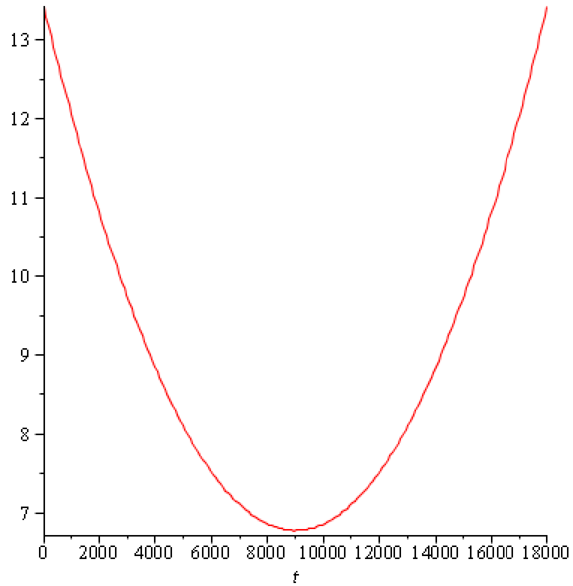

$S$



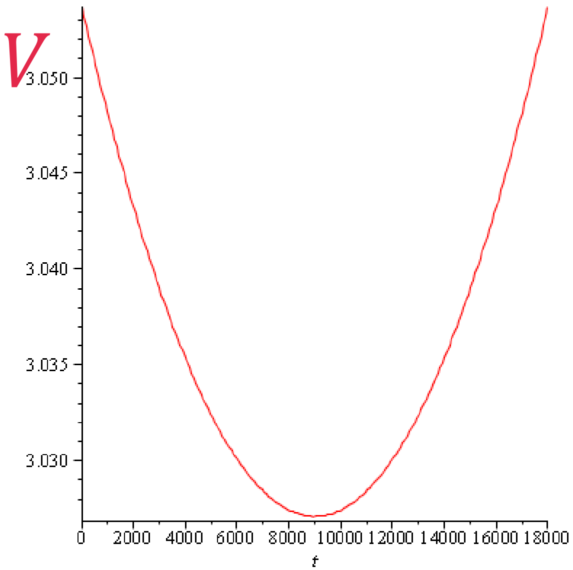

$p$




$\dot{S}$




$V$



# Advantages of the modern theory

- Relaxation in the smoothness requirements for $L(t, S, V)$ and $S(t)$ to account for non-smoothness in price, battery models
- Natural incorporation of constraints and penalties

- But possibly including unphysical solutions, mathematically oversophisticated, and computations may require classical methods

# Modern theory of CoV (after Rockafellar)

- Functional $J = \int_{t_s}^{t_e} L\left(t, S(t), \dot{S}(t)\right)\, dt + \ell\big(S(t_s), S(t_e)\big)$ where $L$ and $\ell$ may take value $\infty$ to incorporate constraints
- Technical assumptions:
  - $\ell$ is lower semi continuous (lsc), proper
  - $L$ is lsc, proper, a normal integrand and
    - $L(t, S, V) \geq a(t, S, V)$ a mild growth condition
  - $L(t, S, V)$ is convex in $S, V$
  - The path $S(t)$ is absolutely continuous, $\dot{S}(t)$ exists a.e. and $\dot{S} \in \mathcal{L}^1$

# Modern theory

- Theorem. Suppose $S(t)$ is a path with $J[S\} < \infty$ and suppose $Y(t)$ is a path satisfying the generalised Euler-Lagrange condition $\left(\dot{Y}(t), Y(t)\right) \in \partial_{S,V} L(t, S(t), \dot{S}(t))$ for a.a. $t$ and also the generalised transversality condition $\left(Y(t_s), -yY(t_e)\right) \in \partial\ell\left(S(t_s), S(t_e)\right)$ then $x(t)$ is optimal.

- Note $\partial_{X,V} L(t, S(t), \dot{S}(t))$ and $\partial\ell\left(S(t_s), S(t_e)\right)$ are subgradients

# Extensions

- Develop theory for charging/discharging
- Include e.g. ramp-up penalty say $-\int_{t_1}^{t_2} \left(\frac{dW_R}{dt}\right)^2 dt$
- Include more complicated pricing structures e.g. price for providing power $P_W$
- Consider ensembles of prosumers/battery models
- Design of pricing policy to control system and to incentivize development of storage
- Incomplete information, stochastic pricing

Thank you for your attention