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On 29 January 2001 Gennadii Vladimirovich Belyi died suddenly in Vladimir.

He was born on 2 February 1951 in Magnitogorsk and his parents moved to the Dnepropetrovsk district in Ukraine. In 1968 he left the Kiev Physics and Mathematics boarding school and entered the Department of Mechanics and Mathematics of Moscow State University. After graduation he worked from 1973 to 1975 in Kiev and Lvov. From 1975 to 1978 he was a PhD student at the Steklov Mathematical Institute of the Academy of Sciences of the USSR, where his research supervisor was I. R. Shafarevich. In 1979 he defended his candidate's dissertation, and a year before that he started teaching in the higher mathematics group at Vladimir State University, first as an assistant, then as a senior lecturer, and since 1982 as a docent.

His major publications are devoted to algebra and number theory, and above all to the Galois theory of algebraic number fields.

For some years after his graduation he concentrated on the problem of constructing field extensions with simple Galois groups over the rational field $\mathbb{Q}$ and cyclotomic fields, starting from the known construction of such extensions over $\mathbb{C}(T)$ and $\overline{\mathbb{Q}}(T)$, where $\mathbb{C}$ is the complex field and $\overline{\mathbb{Q}}$ is the field of all algebraic numbers. After many years' effort he found a very simple condition for a finite group G to be realizable as a Galois group over a cyclotomic field. This condition concerns the existence of a generating set $g_1, \dots, g_k$ for G with the relation $g_1 \dots g_k = 1$, such that any elements $g'_1, \dots, g'_k$ separately conjugate to $g_1, \dots, g_k$ and satisfying the same relation can be obtained simultaneously from $g_1, \dots, g_k$ by conjugation by a single element of G. This condition can be verified effectively and it was used to prove that a whole class of finite simple groups are Galois groups of extensions of cyclotomic fields. In some cases these groups can also be realized as Galois groups of extensions over $\mathbb{Q}$. This constituted an unexpected breakthrough in the inverse Galois problem. In the next few years he showed that his condition is satisfied by most simple groups of Lie type over finite fields, so that they are Galois groups of extensions of cyclotomic fields. This theme was taken up by other authors, who significantly widened the range of applicability of these ideas. The inverse Galois problem has now been solved by this method over cyclotomic fields for nearly all finite simple groups, and for some of them (for example, for the famous 'Monster' group)[1] it has also been solved over $\mathbb{Q}$.

This work constituted Belyi's candidate's dissertation. It was so highly regarded during the defence of the dissertation that awarding the doctoral degree was discussed. Unfortunately, the formalities made this impossible.

In the same work, using very simple considerations (on just one page), Belyi deduced an unexpected criterion for an algebraic curve to be defined over some algebraic number field (equivalently, over $\overline{\mathbb{Q}}$). The criterion is that the curve is a cover of the projective line with three branch points.

It is interesting that this criterion was simultaneously formulated as a conjecture by A. Grothendieck, for whom it played an important role in his combinatorial approach to the theory of the moduli variety of algebraic curves, known as 'dessins d'enfants'. In his memoirs Grothendieck writes that when he mentioned this conjecture to P. Deligne he replied that it had just been proved by Belyi, who was unaware of Grothendieck's conjecture. Grothendieck writes "never have so many ideas been described in so few lines" as in this proof of Belyi.

This result of Belyi, which became known as the three-points theorem, has been applied in very diverse areas of algebraic geometry in recent years. We mention just some of the applications. Developing the method of his proof, F. A. Bogomolov and T. Pantev have found a very simple proof of the existence of a smooth projective model for every algebraic variety over a field of characteristic zero.[2] Belyi's method allowed him to embed the Galois group of $\overline{\mathbb{Q}}$ over $\mathbb{Q}$ into the automorphism group of a free group of rank 2. Furthermore, this embedding led to the discovery of highly non-trivial connections with the automorphism group of the braid group (V. G. Drin'feld, Y. Ihara).

Belyi demonstrated a remarkable degree of persistence. He worked at problems for years, although everybody was convinced that his ideas were not sufficient to solve the problems. And as a result he found new ideas which led him to solutions.

Recently he worked on the so-called 'Markov spectrum' or 'Markov numbers'. Here the situation looked similar to his work on Galois theory: as though his enormous mathematical potential was not adequate for the problems in this area. But he persisted. And one may speculate that if he had been granted a few more years of life he might have been proved correct.

The outstanding contribution of Belyi to algebra and number theory has been recognised by mathematicians worldwide. He was awarded a Moscow Mathematical Society prize in 1981. He was an invited speaker at the ICM in Berkeley (USA) in August, 1986. In the 1990s he took part in many conferences and worked in a large number of mathematical institutes (Chicago, Heidelberg, Göttingen, Berlin, Bonn).

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[1] See the survey in *D. Hilbert*, *Selected Works*, vol. 1, Faktorial, Moscow 1998, pp. 522–523 (Russian).

[2] "Weak Hironaka theorem", *Math. Res. Lett.* **3** (1996), 299–308.

The friends and colleagues of Gennadii Vladimirovich Belyi are shaken by the news of his unexpected death. His untimely departure is an enormous loss to everyone who knew him.

F. A. Bogomolov, N. I. Dubrovin, V. A. Iskovskikh, V. S. Kulikov, A. N. Parshin, I. R. Šhafarevich

Publications of G. V. Belyi

[1] with V. A. Korolevich, "Serre Lie algebras of generalized Jacobians", *Mat. Zametki* **19** (1976), 571–576; English transl., *Math. Notes* **19** (1976), 347–349.

[2] "On Galois extensions of the maximal cyclotomic field", *Izv. Acad. Nauk SSSR Ser. Mat.* **43** (1979), 267–276; English transl., *Math. USSR-Izv.* **14** (1980), 247–256.

[3] "On extensions of the maximal cyclotomic field having a given classical Galois group", *J. Reine Angew. Math.* **341** (1983), 147–156.

[4] "The commutator subgroup of the absolute Galois group", Proc. International Congress of Mathematicians (Berkeley, California, 1986), Amer. Math. Soc., Providence, RI 1987, pp. 346–349; English transl., *Amer. Math. Soc. Transl. Ser.* 2 **147** (1990), 21–24.

[5] "Another proof of the three-points theorem", Preprint MPI-1997-46.

[6] "Markoff's numbers and quadratic forms", Preprint MPI-1997-47.

[7] Russian translation of D. Hilbert's paper, "Über die Irreduzibilität ganzer rationaler Funktionen mit ganzzahligen Koeffizienten" (*J. Reine Angew. Math.* **110** (1892), 104–129), *D. Hilbert*, *Selected works*, vol. 1, Faktorial, Moscow 1998, pp. 128–147.

[8] "Markov's numbers and quadratic forms", (Proc. Internat. Conf. dedicated to the 90th birthday of L. S. Pontryagin, vol. 8 (Algebra)), Itogi Nauki i Tekhniki: Sovr. Mat. i Prilozhen., vol. 69, VINITI, Moscow 1999, pp. 5–23; English transl., *J. Math. Sci.* **106** (2000), 3087–3097.

[9] "Another proof of the three-point theorem", *Mat. Sb.* **193** (2002), 21–24; English transl., *Sbornik: Math.* **193** (2002), 329–332.

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