2018 LTCC Course on **Aperiodic Order**

Worksheet 2

Consider the inflation rule

$$\varrho : \begin{array}{l} a \mapsto abb \\ b \mapsto a \end{array}$$

on the two-letter alphabet $\{a, b\}$.

Exercise 1: Show that ϱ is irreducible and primitive.

Exercise 2: Consider the four possible two-letter seeds $a|a$, $a|b$, $b|a$ and $b|b$. Which of these are legal? Iterate ϱ on the legal seeds. Are there any fixed points under ϱ? Find all fixed points under ϱ^2.

Exercise 3: Calculate the corresponding substitution matrix M and its leading eigenvalue λ. If you start from the initial word $w^{(0)} = a$ and define $w^{(n+1)} = \varrho(w^{(n)})$, show that $w^{(n+1)} = w^{(n)}w^{(n-1)}w^{(n-1)}$. How many letters of each type does the word $w^{(8)}$ have?

Exercise 4: Calculate the right and left eigenvector of M to the eigenvalue λ. What is the frequency of letters a and b in a fixed point word? Compare this with the result for $w^{(10)}$ obtained above.

Exercise 5: If the short interval is chosen to have length 1, what is the length of the long interval in the corresponding geometric inflation rule? Consider the point set $\varLambda$ of all left interval endpoints, and calculate the average distance of points. What is the density of the point set $\varLambda$?