2018 LTCC Course on **Aperiodic Order**

Worksheet 1

Exercise 1: For each of the point sets in $\mathbb{R}$ below, decide whether they are relatively dense, uniformly discrete, Delone, FLC and Meyer. Here, $\mathbb{N} = \{n \in \mathbb{Z} \mid n > 0\}$ denotes the natural numbers.

(a) $\varLambda_a = 2\,\mathbb{Z}$;

(b) $\varLambda_b = \{n + 1/n \mid n \in \mathbb{Z} \setminus \{0\}\}$;

(c) $\varLambda_c = -\mathbb{N} \cup \{0\} \cup \sqrt{3}\,\mathbb{N}$;

(d) $\varLambda_d = \mathbb{Z} \setminus S$, where S is an arbitrary subset of $2\,\mathbb{Z}$.

Exercise 2: Calculate the cyclotomic polynomials $Q_\ell(x)$ for $1 \leq \ell \leq 6$.

Exercise 3: Consider the set $\mathbb{Z}[\xi] = \{a_0 + a_1\xi + a_2\xi^2 + a_3\xi^3 \mid a_0, a_1, a_2, a_3 \in \mathbb{Z}\}$, where $\xi = \exp(2\pi\mathrm{i}/5)$ is a primitive fifth root of unity. Show that $\mathbb{Z}[\xi]$, when interpreted as a point set in $\mathbb{R}^2$, is invariant under rotations by $\pi/5$.

Exercise 4: Show that $\mathbb{Z}[\tau] = \{m + n\tau \mid m, n \in \mathbb{Z}\}$, where $\tau = (1 + \sqrt{5})/2$ is the golden ratio, satisfies $\tau\mathbb{Z}[\tau] = \mathbb{Z}[\tau]$.

Exercise 5: Consider the ring of numbers $\mathbb{Z}[\sqrt{2}] = \{m + n\sqrt{2} \mid m, n \in \mathbb{Z}\}$. Using the algebraic conjugation $\sqrt{2} \mapsto -\sqrt{2}$ in $\mathbb{Q}(\sqrt{2})$, construct the Minkowski embedding of $\mathbb{Z}[\sqrt{2}]$.