Residue Theorem

Evaluate $\int_C \frac{e^z}{z^4-1}\,dz$ where $C = \{z : |z| = 2\}$

$$f(z) = \frac{e^z}{z^4-1}$$

$$z^4 - 1 = 0$$

ie $z = \pm 1, \pm i$

$$f(z) = \frac{g(z)}{h(z)}$$

$$h(\alpha) = 0, \ h'(\alpha) \neq 0$$

$$\text{Res}(f, \alpha) = \frac{g(\alpha)}{h'(\alpha)}$$

$$h(z) = z^4 - 1$$

$$h'(z) = 4z^3$$

$$g(z) = e^z$$

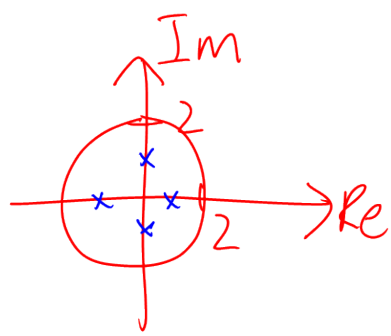
$$\text{Res}(f, 1) = \frac{g(1)}{h'(1)} = \frac{e^1}{4} = \underline{\underline{\frac{1}{4}e}}$$

$$\operatorname{Res}(f,-1) = \frac{e^{-1}}{4(-1)^3} = \underline{\underline{-\frac{1}{4}e^{-1}}}$$

$$\operatorname{Res}(f,i) = \frac{e^{i}}{4(i)^3} \times \frac{i}{i} = \underline{\underline{\frac{ie^{i}}{4}}}$$

$$\operatorname{Res}(f,-i) = \frac{e^{-i}}{4(-i)^3} = \underline{\underline{-\frac{1}{4}ie^{-i}}}$$

$$\int_C \frac{e^{z}}{z^4-1}\,dz \qquad C = \{z : |z| = 2\}$$



$$\int_C f(z)\,dz = 2\pi i S$$

$$\int_C \frac{e^z}{z^4-1}\,dz = 2\pi i\left(\frac{1}{4}e^1-\frac{1}{4}e^{-1}+\frac{1}{4}ie^i-\frac{1}{4}ie^{-i}\right)$$

$$\sinh x = \frac{e^x-e^{-x}}{2}$$

$$= 2\pi i\left[\frac{1}{2}\sinh 1+\frac{1}{2}i\sinh i\right]$$

$$= \pi i(\sinh 1 + i\sinh i)$$

$$i\sin x = \sinh ix$$

$$= \pi i(\sinh 1 + i \times i\sin 1)$$

$$= \pi i(\sinh 1 - \sin 1)$$