Techniques of Fractal Geometry, Kenneth J Falconer, Wiley 1997

This book has proved surprisingly hard to write a review for. This is not due to problems with the book (indeed it is excellent) but more due to the reviewer's concerns with his own bias: he was involved in the proof-reading.

This book is a sequel to the author's earlier book, *Fractal Geometry* [1] (recently published by Wiley in paperback). It follows closely the style of its ancestor and details in a clear and elegant manner developments in fractal geometry since the earlier book was published. The author concentrates on presenting the key ideas in the subject rather than obtaining the most general results. This makes the book a useful primer before reading more technical papers/works.

The book begins by reviewing the some of the more useful results from *Fractal Geometry*, obviating the need for the reader to possess this book as well (although it wouldn't do any harm).

The first part of the book discusses general methods for calculating the dimension (be it Hausdorff, packing or Box) of a set and introduces the family of examples which the book is primarily concerned with: cookie-cutter sets. These are essentially non-linear versions of the more familiar self-similar sets and many of the results for self-similar sets hold for this more general class. The thermodynamic formalism is introduced and it is shown how this enables dimensions of these non-linear sets to be (theoretically) found. The author explains clearly the analogies with thermodynamics and makes the methods used appear very natural.

The middle part of the book describes how probabilistic ideas may be used in the study of fractals. In particular, the author states and proves versions of the Ergodic theorem, Renewal theorem and the Martingale Convergence theorem. He gives a nice application of the Martingale Convergence theorem to the study of Random cut-out sets. (A set which is formed by randomly cutting out pieces of decreasing size from some initial set, such as a square.) If you know how the size of the cut-out pieces varies then with positive probability you know the dimension of the cut-out set.

The latter part of the book is a survey of several topics from geometric measure theory and fractal geometry, discussing some of the developments of recent years. There is an introduction to the theory of tangent measures and an explanation of their rôle in studying the geometry of measures (and sets) in Euclidean spaces. This chapter serves as a useful primer to the more technical account given by Mattila in [2].

There is a chapter discussing dimensions of measures; this involves, essentially, looking for 'typical' values of

$$\underline{\alpha}(\mu,x)=\liminf_{r\searrow 0}\frac{\mu(B(x,r))}{\log r},\quad \overline{\alpha}(\mu,x)=\limsup_{r\searrow 0}\frac{\mu(B(x,r))}{\log r}$$

for a measure μ (here $B(x,r)$ denotes the closed ball of centre x and radius r). When $\underline{\alpha}$ (or $\overline{\alpha}$) may take many different values we are led naturally to the idea of decomposing measures into pieces where they do behave similarly. This in turn leads, ultimately, to investigating the multifractal decomposition of a measure which involves looking at the dimensions of sets of the form

$$\{x:\underline{\alpha}(\mu,x)=\alpha\}\ \text{ or }\ \{x:\overline{\alpha}(\mu,x)=\alpha\}\,.$$

The final chapter of the book gives a brief but lively introduction to the role of fractals in the study of differential equations including discussions of how to estimate dimensions of attractors, and how to solve the heat equation on domains with a nice self-similar boundary. This involves a nice application of the Renewal theorem discussed earlier in the book.

To summarise: this is a lovely book and a worthy successor to Fractal Geometry. It presents concisely many of the techniques currently used in the study of fractals and is a good survey of results which have appeared since the publication of the last book.

The book, unfortunately, does possess many minor errors which may be occasionally distracting (this is especially embarrassing to this reviewer since he was involved with proof-reading…) but even the most serious error I found (in the chapter on tangent measures, where the set of tangent measures of $\log 2/\log 3$-dimensional Hausdorff measure restricted to the usual $\frac{1}{3}$-Cantor set is misdescribed) is not particularly bad.

The author's clear, lucid style of writing is a pleasure to read and his decision to concentrate on the essential ideas rather than obtain the greatest generality pays off handsomely. Anyone who reads this book will, by the end of it, be able to pick up a current research paper on fractals and have a good chance of understanding it.

References

[1] K. J. Falconer, *Fractal Geometry*, Wiley, 1989.

[2] P. Mattila, *Geometry of Sets and Measures in Euclidean Spaces*, CUP, 1995.

Toby O'Neil,
11 March 1998.