

# Meaning and Dialogue Coherence: A Proof-theoretic Investigation

Paul Piwek

Centre for Research in Computing  
The Open University  
Milton Keynes, UK  
p.piwek@open.ac.uk

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## Abstract

This paper presents a novel proof-theoretic account of dialogue coherence. It focuses on cooperative information-oriented dialogues and describes how the structure of such dialogues can be accounted for in terms of a multi-agent hybrid inference system that mixes natural deduction with information transfer and observation. We show how the structure of dialogue arises out of the interplay between the inferential roles of logical connectives (i.e., sentence semantics), a rule for transferring information between agents, and rules for information flow between agents and their environment. Our order of explanation is opposite in direction to that adopted in the game-theoretic semantics tradition, where sentence semantics (or a notion of valid inferences) is derived from (winning) dialogue strategies. The approaches may, however, be reconcilable, since we focus on cooperative dialogues, whereas the latter concentrates on adversarial dialogue.

## 1 Introduction

Models of coherence come in many different shapes, from proposals based on scripts, grammars, and social rule following to models of topic continuity. A now slightly dated collection that provides an overview of the multitude of approaches to *dialogue coherence* is Craig and Tracy (1983). More recently, Mann (2002) surveys a number of extant analyses of dialogue coherence.

The aim of this paper is to work out in detail a notion of coherence for one particular type of cooperative dialogues, rather than to criticize or

dismiss other approaches. In our view, coherence is a complex phenomenon that is likely to require analyses from more than one single perspective.

We provide an explication of dialogue coherence in terms of the meaning of the expressions that are used in a dialogue against the background of the participants' discursive dispositions. Thus, coherence is modelled as a property of dialogues whose meaning-bearing parts fit together in a certain way in context. Our analysis of dialogue coherence will only provide the foundations for certain aspects of dialogue coherence in general. At the end of this section, we specify the precise scope of the current proposal.

To construct an explication of dialogue coherence along these lines, we adopt the following strategy. Firstly, we describe a theory of meaning that provides the foundation for the current endeavour. This theory complies with the Wittgensteinian slogan that "meaning is use".<sup>1</sup> This pragmatist slogan is fleshed out by identifying the meaning of an expression with its role in reasoning. This role is given by the circumstances of appropriate *application* of the expression and the appropriate *consequences* of such an application. The meaning of logical vocabulary will be assigned a privileged status in this undertaking and receive a formalization in terms of a variant of Gentzen's (1934) calculus of Natural Deduction. Secondly, this standard Natural Deduction calculus for solitary reasoners is extended to a calculus for *multiple situated* reasoners. Thirdly, this extended Natural Deduction calculus is used to model dialogue coherence. Whereas Gentzen's

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<sup>1</sup>Note that in an important respect our investigation is not Wittgensteinian; we do not share the later Wittgenstein's skepticism about the possibility of rigorous theories of language use.

calculus allows us to characterize valid inferences, the extended calculus demarcates a certain type of coherent dialogue. We provide examples of dialogues that are generated by the calculus and use these to bolster the initial plausibility of the claim that coherence according to the extended calculus mirrors coherence of natural language dialogue. This is achieved by drawing attention to a number of structural properties of naturally occurring dialogues that are also found in dialogues that are generated with the extended calculus.

Walton and Krabbe (1995) point out that dialogue comes in many varieties. Each variety has its own distinctive purpose and participant aims and, as a result, concomitant notion of coherence. We do not intend to define coherence regardless of dialogue type, but rather restrict our attention to a specific type of dialogue, which we call the *cooperative information-oriented dialogue*. The *main purpose* of this type of dialogue is the exchange of information. The *participants' aim* is to cooperate with each others' requests for information; in particular, no persuasion, negotiation or coercion is required. Cooperative information-oriented dialogues have been a central topic of study in computational linguistics and natural language processing, e.g., witness the large number projects on dialogue systems for providing travel information.

Finally, a remark on the theoretical orientation of this paper. It is not concerned with directly accounting for instances of naturally occurring dialogues. Rather, our aim is to provide an abstract model of cooperative information-oriented dialogue that captures and accounts for certain abstract patterns that have been observed in naturally occurring dialogues by conversation analysts (Sudnow, 1972). We discuss these patterns, specifically adjacency pairs and insertion sequences, in more detail further on in this paper.

## 2 Meaning as Inferential Role

The theory of meaning that we employ follows broadly the meaning-theoretic deliberations of Brandom (1994). The formalization is along the lines described in Sundholm (1986),<sup>2</sup> though there are also important differences (see section 7). Meaning is characterized in terms of inferential

<sup>2</sup>Sundholm bases much of his formalization on proposals by the philosophers/logicians Michael Dummett and Dag Prawitz.

role, rather than truth-conditions. For the purpose of this paper, no specific theory of truth is put forward; we get by without that notion. This does, however, not exclude the possibility of a reconstruction of truth in terms of the framework described in this paper. Cf. Brandom (1994).

We start with a framework involving a single agent, henceforth  $\alpha$ . Our system captures the practical ability of this agent to reason with expressions of a language  $\mathcal{L}$ . This language consists exclusively of atomic formulae  $At \subset \mathcal{L}$ , and formulae that are constructed from formulae in  $\mathcal{L}$  using the connectives for implication ' $\rightarrow$ ' and conjunction '&': if  $A, B \in \mathcal{L}$ , then  $(A \rightarrow B) \in \mathcal{L}$  and  $(A \& B) \in \mathcal{L}$ .<sup>3</sup>

Inferences are formalized in terms of *judgements* of the form  $[\alpha] H \vdash A$ . These should be read as agent  $\alpha$  (henceforth, references to agents are omitted when it is clear from the context which agent the judgement belongs to) affirms/derives  $A$ , given the temporary assumptions  $H$  (i.e., assumptions that are only accessible for the duration of the inference). In addition to the collection of temporary assumptions ( $H$ ), an agent, such as  $\alpha$ , also relies on a set of persistent assumptions ( $\Gamma_\alpha$ ). In our system,  $\Gamma_\alpha$  functions like a global variable in a programming language whose value is accessible any time during an inference. The value of  $\Gamma_\alpha$  can be updated through declarations, as is common for global variables. For instance, the following declaration adds the proposition letter  $a$  to  $\Gamma_\alpha$ 's current value:  $\Gamma_\alpha := \Gamma_\alpha \cup \{a\}$ . Note that we use capitals (e.g.,  $A$  and  $B$ ) as meta-variables over proposition letters and lower case (e.g.,  $a$  and  $b$ ) for the actual proposition letters.

An assumption  $A \in (H \cup \Gamma_\alpha)$  is thought of in terms of the disposition of  $\alpha$  to affirm  $A$ . This disposition is made explicit by the following deduction rule:

$$(1) \text{ (member)} \quad \frac{A \in \Gamma \cup H}{H \vdash A}$$

This rule says that formula  $A$  can be inferred/derived/deduced from  $H$  and the implicit persistent assumptions  $\Gamma$ , if  $A$  is a member of the union of  $\Gamma$  and  $H$ . The set  $\Gamma \cup H$  plays an inferential role in this system. This inferential role takes the place of a classical explication in terms of representation/truth-conditions.

<sup>3</sup>Henceforth, brackets will be omitted when there is no danger of ambiguity.

The inferential role of logical vocabulary is given a special place in the current scheme: a logical connective allows an agent to formulate explicitly a pattern of inference that it already follows. For instance, an agent who is disposed to deriving ‘The tiles get wet’ from ‘it rains’, can make this practical activity explicit by affirming ‘If it rains, the tiles get wet’.

The meaning of the logical connective is given by the circumstances of appropriate application of that connective and the appropriate consequences of such an application. For the conditional ‘ $\rightarrow$ ’ the appropriate circumstances of application are given by the following rule for introducing a conditional:

$$(2) \text{ (arrow intro)} \quad \frac{H \cup \{A\} \vdash B}{H \vdash A \rightarrow B}$$

Thus, we can derive  $A \rightarrow B$ , if we can derive  $B$  from our assumptions extended with  $A$ . The appropriate consequences of using ‘ $\rightarrow$ ’ are given by the following rule for eliminating ‘ $\rightarrow$ ’:

$$(3) \text{ (arrow elim)} \quad \frac{H \vdash A \rightarrow B \quad H \vdash A}{H \vdash B}$$

This rule is chosen so that the arrow intro and elim rules together introduce only inferences regarding the logical connective ‘ $\rightarrow$ ’. In Dummett’s terms, the rules are in harmony with antecedent inferential practices. This requirement is essential, because of the explicative role of logical vocabulary: it should serve to make explicit existing inferential practices; it should not license novel inferences involving the pre-existing vocabulary, since that would destroy its explicative role with respect to that pre-existing vocabulary.

The rules for conjunction introduction and elimination are the following:

$$(4) \text{ (conj. intro)} \quad \frac{H \vdash A \quad H \vdash B}{H \vdash A \& B}$$

$$(5) \text{ (conj. elim)} \quad \frac{H \vdash A \& B}{H \vdash A} \quad \frac{H \vdash A \& B}{H \vdash B}$$

### 3 System $S_1$ : Situated Inferential Practice and Dialogue

The system presented so far is limited to solitary reasoners that are isolated both from other reasoners and the world around them. In this section, we present an extension which removes the former limitation. We will refer to the system described in the current section as  $S_1$ .

#### 3.1 The Transfer Rule

We introduce a set of agents  $\mathcal{A}$ . We use  $\alpha, \beta, \gamma, \dots$  as meta-variables over members of  $\mathcal{A}$ . We can now add a rule for *transferring* proof goals between agents:

$$(6) \text{ (tr)} \quad \frac{[\beta] H \vdash A}{[\alpha] H \vdash A} \quad \Gamma_\alpha := \Gamma_\alpha \cup \{\bigwedge H \rightarrow A\} \text{ and } \langle \alpha, \beta \rangle \in \mathcal{C}$$

This transfer rule (tr) tells us that if agent  $\beta$  can derive  $A$  under the assumptions in  $H$ , then agent  $\alpha$  can also derive  $A$  under the assumptions in  $H$ , provided that the two side conditions (given on the right-hand side) are satisfied. The first condition says that the context  $\Gamma_\alpha$  of assumptions entertained by  $\alpha$ , should be extended with  $\bigwedge H \rightarrow A$ . Here,  $\bigwedge H$  stands for the conjunction  $A_1 \& A_2 \& \dots$  of the formulae  $A_1, A_2, \dots$  that are members of  $H$ . If  $H$  is empty,  $\bigwedge H \rightarrow A = A$ . The second condition ( $\langle \alpha, \beta \rangle \in \mathcal{C}$ ) says that there should be a transfer channel between  $\alpha$  and  $\beta$  (where  $\mathcal{C} \subseteq \mathcal{A} \times \mathcal{A}$ ). We use  $\mathcal{C}$  in combination with this side condition to model situations in which not every agent can exchange information with every other agent in  $\mathcal{A}$ . However, unless stated otherwise, we will henceforth assume that  $\mathcal{C} = \mathcal{A} \times \mathcal{A}$ , i.e., information can be transferred between any pair of agents.

#### 3.2 Example of a Proof Tree

Take a situation involving the agents  $\alpha, \beta$  and  $\gamma$  in which  $\Gamma_\alpha = \emptyset$ ,  $\Gamma_\beta = \{a\}$ , and  $\Gamma_\gamma = \{b\}$ . Let us assume that  $\alpha$  wants to build a proof for  $a \& b$ . Since neither  $a$  nor  $b$  is part of  $\Gamma_\alpha$ ,  $\alpha$  will need to access information held by  $\beta$  and  $\gamma$ . The following proof tree illustrates how exactly:

$$(7) \quad \frac{\frac{[\beta] a \in \Gamma_\beta \cup \emptyset}{[\beta] \emptyset \vdash a} \text{ (mem.)} \quad \frac{[\gamma] b \in \Gamma_\gamma \cup \emptyset}{[\gamma] \emptyset \vdash b} \text{ (mem.)}}{\frac{[\beta] \emptyset \vdash a}{[\alpha] \emptyset \vdash a} \text{ (1)(tr)} \quad \frac{[\gamma] \emptyset \vdash b}{[\alpha] \emptyset \vdash b} \text{ (2)(tr)}}{[\alpha] \emptyset \vdash a \& b} \text{ (conj.intro)}$$

SIDE CONDITIONS: (1)  $\Gamma_\alpha := \Gamma_\alpha \cup \{a\}$ ; (2)  $\Gamma_\alpha := \Gamma_\alpha \cup \{b\}$ .

Note that we omitted the side condition regarding the transfer channel. We conveniently assumed that all agents can exchange information with all other agents.

Execution of the two side conditions results in  $\Gamma_\alpha = \{a, b\}$ . As a result of the construction of this

proof, we have arrived at a  $\Gamma_\alpha$  in which a proof for  $a\&b$  can be constructed directly, without recourse to the transfer. In other words,  $a\&b$  has become part of  $\alpha$ 's information.

### 3.3 From Proof Trees to Dialogue Structure

The last stage consists of the transformation of the proof tree to a dialogue (structure). Here we provide an outline of the algorithm, which has been fully implemented.<sup>4</sup> We proceed in two steps. Firstly, we map the hierarchical tree to a linear structure where each tree node is represented by an item in the linear structure. (e.g., items 4 and 9 below each represent a single node; the nodes in question are the terminal nodes of tree 7), and possibly a second item indicating that the part of the tree dominated by the node has been closed (e.g., the pairs  $\langle 1, 12 \rangle$  and  $\langle 2, 6 \rangle$  represent single tree nodes; the former corresponds to the root node of tree 7). For the tree in 7, we obtain the following linear representation:

- (8)
1.  $\alpha$  : goal-know-if( $a\&b$ )
  2.  $\alpha$  : (transfer) goal-know-if( $a$ )
  3.  $\beta$  : goal-know-if( $a$ )
  4.  $\beta$  : in-assumptions( $a$ )
  5.  $\beta$  : confirmed( $a$ )
  6.  $\alpha$  : confirmed( $a$ )
  7.  $\alpha$  : (transfer) goal-know-if( $b$ )
  8.  $\gamma$  : goal-know-if( $b$ )
  9.  $\gamma$  : in-assumptions( $b$ )
  10.  $\gamma$  : confirmed( $b$ )
  11.  $\alpha$  : confirmed( $b$ )
  12.  $\alpha$  : confirmed( $a\&b$ )

For brevity's sake, we have omitted reference to the empty set of temporary hypotheses. Strictly speaking we should, for instance, have written goal-know-if( $a\&b$ ,given-that, $\emptyset$ ) instead of goal-know-if( $a\&b$ ).

The sequence in 8 is not yet a straightforward dialogue. It contains various locutions which can be thought of as internal monologues of the interlocutors with themselves, but not actual dialogue locutions. For example 6.  $\alpha$  : confirmed( $a$ ), is superfluous after 5.  $\beta$  : confirmed( $a$ ).  $a$  can be taken to have been confirmed by  $\alpha$  implicitly, simply by  $\alpha$  proceeding with the dialogue.

For the mapping from an extensive dialogue representation, such as 8, to a more economic dialogue structure we use the following rules:

- $\alpha_i$  : goal-know-if( $A$ )  $\mapsto$   $\alpha_i$  : I am wondering whether  $A$ .
- $\alpha_i$  : (transfer) goal-know-if( $A$ ),  $\alpha_j$  : I am wondering whether  $A$   $\mapsto$   $\alpha_i$  : Tell me  $\alpha_j$ ,  $A$ ?
- $\alpha_i$  : confirmed( $A$ ),  $\alpha_j$  : confirmed( $A$ )  $\mapsto$   $\alpha_i$  : confirmed( $A$ ).
- $\alpha_i$  : in-assumptions( $A$ ),  $\alpha_i$  : confirmed( $A$ )  $\mapsto$   $\alpha_i$  :  $A$ .
- $\alpha_i$  : confirmed( $A$ )  $\mapsto$   $\alpha_i$  : That confirms  $A$ .

When these mapping rules are applied to 8, we obtain:

- (9)
1.  $\alpha$  : I am wondering whether  $a\&b$ .
  2.  $\alpha$  : Tell me  $\beta$ ,  $a$ ?
  3.  $\beta$  :  $a$ .
  4.  $\alpha$  : Tell me  $\gamma$ ,  $b$ ?
  5.  $\gamma$  :  $b$ .
  6.  $\alpha$  : That confirms  $a\&b$ .

Note that this dialogue structure exhibits two well-known conversation analytical configurations (Sudnow, 1972): the adjacency pairs (2,3), (4,5) and (1,6) and the insertion sequence (2,3,4,5).

## 4 Generative Systems as Abstract Models of Dialogue

Before we proceed with presenting some extensions to the system  $\mathcal{S}_1$ , let us take a step back and make explicit what such systems have in common. Each of them functions as an abstract model of cooperative information-oriented dialogue, and has the following components:

1. A hybrid inference system  $\mathcal{I}$  consisting of:
  - (a) A language  $\mathcal{L}$  (e.g., the language of propositional logic or a fragment thereof);
  - (b) A set of agents  $\mathcal{A}$ , each with a set of assumptions  $\Gamma_{\alpha_i}$ ;
  - (c) A communication channel  $\mathcal{C}$  that specifies which agents can communicate with what other agents (i.e.,  $\mathcal{C} \subseteq \mathcal{A} \times \mathcal{A}$ );
  - (d) A set of hybrid inference rules  $\mathcal{R}$  for the language and the agents. The rules are hybrid because they can encompass natural deduction, observation and communication. The rules enable us to build proof trees (or proof search trees, as we will see in a moment).

<sup>4</sup>See [mcs.open.ac.uk/pp2464/resources](http://mcs.open.ac.uk/pp2464/resources)

2. A specification of the set of potential dialogues  $\mathcal{D}_P$  between the agents, given the language  $\mathcal{L}$ .
3. A mapping  $m$  from proof trees, generated with  $\mathcal{I}$ , to coherent dialogues  $\mathcal{D}$ .

In short, a generative system  $\mathcal{S}$  is a tuple of the form  $\langle \mathcal{I}, \mathcal{D}_P, m \rangle$ . The purpose of such a system is the characterization of coherent dialogues (members of  $\mathcal{D}$ ). This is achieved by using  $\mathcal{I} = \langle \mathcal{L}, \mathcal{A}, \mathcal{C}, \mathcal{R} \rangle$  to generate proof trees, that is trees representing valid inferences (or searches for valid inferences). These proof trees are then mapped by  $m$  to members of  $\mathcal{D}_P$  (the set of potential dialogues). A member of  $\mathcal{D}_P$  that can be generated from a proof tree using  $m$  is a member of the set of proper, i.e., coherent, dialogues  $\mathcal{D}$  (with  $\mathcal{D} \subset \mathcal{D}_P$ ). The mapping  $m$  basically turns a hierarchical proof tree into a linear dialogue representation (omitting most proof steps that do not involve communication between agents).

We investigate systems for the generation of abstract representations of coherent dialogues. The adequacy of such systems can be thought of in terms of their correctness and completeness:

- **CORRECTNESS:** Each member of  $\mathcal{D}$  generated by  $\mathcal{S}$  should represent the structure of a coherent dialogue. Here, coherence is understood roughly speaking in terms of our pre-theoretical understanding of dialogue coherence.
- **COMPLETENESS:** 1. **GLOBAL:** Each structure of a coherent dialogue (again, we refer to our pre-theoretical insight into dialogue coherence) should be generated by  $\mathcal{S}$ , that is, it should be a member of  $\mathcal{D}$ . 2. **LOCAL:** Each structure of a coherent dialogue that is a member of  $\mathcal{D}_P$  should be generated by  $\mathcal{S}$ , that is, it should be a member of  $\mathcal{D}$ .

These notions of correctness and completeness are to guide the construction of the generative systems. For each system, we will attempt to satisfy correctness. As we construct further systems, the main aim will be to add features that make the new system better approximate either local or global completeness.

## 5 System $\mathcal{S}_2$ : From Proof Trees to Proof Search Trees

System  $\mathcal{S}_1$  has one major drawback: it only allows for dialogues generated from completed proof

trees. What is lost, is the *search* for a proof which many cooperative information-oriented dialogues revolve around. In short, System  $\mathcal{S}_1$  is not globally complete: people in conversation will often explore unfruitful paths, and have to use locutions such as: ‘I could not resolve the question whether  $A$ ?’ or ‘I don’t know whether  $A$ ’. A first step toward remedying this situation is the addition of such locutions to  $\mathcal{D}_P$ . Let us be precise, and spell out the set of potential dialogues  $\mathcal{D}_P$  using BNF notation:

$$(10) \begin{aligned} \langle D_P \rangle &::= \langle Loc \rangle, \langle D_P \rangle \mid \epsilon \\ \langle Loc \rangle &::= \langle Agent \rangle: \text{ I am wondering} \\ &\quad \text{whether } \langle Prop \rangle \mid \langle Agent \rangle: \text{ Tell me} \\ &\quad \langle Agent \rangle, \langle Prop \rangle? \mid \langle Agent \rangle: \text{ I don't} \\ &\quad \text{know whether } \langle Prop \rangle. \mid \langle Agent \rangle: \\ &\quad \langle Prop \rangle. \mid \langle Agent \rangle: \text{ That confirms} \\ &\quad \langle Prop \rangle. \\ \langle Agent \rangle &::= \alpha \mid \beta \mid \dots \\ \langle Prop \rangle &::= a \mid b \mid \dots \mid \langle Prop \rangle \& \langle Prop \rangle \mid \\ &\quad \langle Prop \rangle \rightarrow \langle Prop \rangle \end{aligned}$$

Now, the problem is one of local completeness: there are now members of  $\mathcal{D}_P$  which are intuitively coherent dialogues (involving the locution ‘ $\langle Agent \rangle$ : I don’t know whether  $\langle Prop \rangle$ ’), but which cannot be generated in  $\mathcal{S}_1$ . To address this problem, we first need to define the mapping  $m$  for proof *search* trees, rather than proof trees.

Let us examine an example of a proof search tree.

$$(11) \frac{\frac{\frac{[\star_1] [\beta] \emptyset \vdash a}{[\alpha] \emptyset \vdash a} \quad \frac{[\gamma] a \in \Gamma_\gamma \cup \emptyset}{[\star_2] [\gamma] \emptyset \vdash a}}{[\alpha] \emptyset \vdash a} \quad \frac{[\gamma] a \in \Gamma_\gamma \cup \emptyset}{[\gamma] \emptyset \vdash a}}{[\alpha] \emptyset \vdash a \& b}}$$

This tree is very similar to the proof tree 7. We have omitted rule labels and conditions to fit the tree on this page. That 11 is a proof *search* tree rather than a proof tree, is indicated by the use of  $\star$ , which marks alternative search branches. Here we have an unsuccessful branch  $\star_1$  and a successful one, i.e.,  $\star_2$  (henceforth we assume that successful search branches are always to the right of unsuccessful ones). This tree would, for example, fit the situation where we initially set out with  $\Gamma_\alpha = \Gamma_\beta = \emptyset$  and  $\Gamma_\gamma = \{a, b\}$ .

As before, we map the tree to a dialogue in two steps. The result of applying the first mapping is:

- (12) 1.  $\alpha$  : goal-know-if( $a\&b$ )
2.  $\alpha$  : (transfer) goal-know-if( $a$ )
3.  $\beta$  : goal-know-if( $a$ )
4.  $\beta$  : not-resolved( $a$ )
5.  $\alpha$  : not-resolved( $a$ )
6.  $\alpha$  : (transfer) goal-know-if( $a$ )
7.  $\gamma$  : goal-know-if( $a$ )
8.  $\gamma$  : in-assumptions( $a$ )
9.  $\gamma$  : confirmed( $a$ )
10.  $\alpha$  : confirmed( $a$ )
11.  $\alpha$  : (transfer) goal-know-if( $b$ )
12.  $\gamma$  : goal-know-if( $b$ )
13.  $\gamma$  : in-assumptions( $b$ )
14.  $\gamma$  : confirmed( $b$ )
15.  $\alpha$  : confirmed( $b$ )
16.  $\alpha$  : confirmed( $a\&b$ )

The second half of the mapping requires the mapping rules of  $\mathcal{S}_1$  and two additional rules:

- $\alpha_i$  : not-resolved( $A$ ),  $\alpha_j$  : not-resolved( $A$ )  $\mapsto$   $\alpha_i$  : not-resolved( $A$ )
- $\alpha_i$  : not-resolved( $A$ )  $\mapsto$   $\alpha_i$  : I don't know whether  $A$ .

Application of the extended set of mapping rules to 12 results in:

- (13) 1.  $\alpha$  : I am wondering whether  $a\&b$ .
2.  $\alpha$  : Tell me  $\beta$ ,  $a$ ?
3.  $\beta$  : I don't know whether  $a$ .
4.  $\alpha$  : Tell me  $\gamma$ ,  $a$ ?
5.  $\gamma$  :  $a$ .
6.  $\alpha$  : Tell me  $\gamma$ ,  $b$ ?
7.  $\gamma$  :  $b$ .
8.  $\alpha$  : That confirms  $a\&b$ .

## 6 System $\mathcal{S}_3$ : Adding Observation

In  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , we went beyond common inference systems, by moving from the model of a solitary reasoner to a community of reasoners (agents) who can exchange information with each other. Communication is, however, not the only way reasoners can acquire new information. In particular, observation of the environment is a further means of information acquisition that the traditional model of logical inference does not deal with. In this respect, our systems  $\mathcal{S}_1$  and  $\mathcal{S}_2$  are also incomplete.

It is beyond the scope of this paper to address all the intricacies of interspersing reasoning with observation. What we do offer is an outline of how

observation can be integrated with multi-agent inference. We explore a minimal extension of  $\mathcal{S}_2$  with the following rule:

$$(14) \text{ (obs.) } \frac{A \in \mathcal{O}_\alpha \quad \text{obs}(\alpha, A)}{[\alpha]H \vdash A} \quad \Gamma_\alpha := \Gamma_\alpha \cup \{A\}$$

This rule states that if the proposition  $A$  is an observable proposition for agent  $\alpha$  (written as  $A \in \mathcal{O}_\alpha$ ) and  $\alpha$  actually observes that  $A$  (i.e.,  $\text{obs}(\alpha, A)$ ), then  $\alpha$  can derive  $A$ . There is one side condition which requires that  $\Gamma_\alpha$  is extended with  $A$ ; that is:  $\Gamma_\alpha := \Gamma_\alpha \cup \{A\}$ .

To give an idea of how the extended calculus can be applied, we model a conversation between  $\pi$  and  $\delta$ .  $\pi$  has the flu and rings  $\delta$  (information desk of  $\pi$ 's surgery) to find out whether she needs to see a doctor ( $sd$ ). For that purpose, she needs to find out whether she has a temperature ( $ht$ ). We have  $\Gamma_\pi = \emptyset$  and  $\Gamma_\delta = \{ht \rightarrow sd\}$ .  $\pi$ 's goal is to find out whether  $sd$ . So  $\pi$  tries to derive relative to her assumptions that  $sd$ . Given  $\Gamma_\pi$ ,  $\pi$  won't succeed unless she decides to communicate. The following is a derivation for  $\pi$  of  $sd$  that involves both communication and observation. It highlights how information possessed by  $\delta$  and observations that only  $\pi$  is able to perform in the situation that we described are combined to obtain a proof for  $sd$ :

$$(15) \frac{\frac{ht \in \mathcal{O}_\pi \quad \text{obs}(\pi, ht)}{[\pi] \emptyset \vdash ht} \quad (3) \text{ (obs.)} \quad \frac{[\delta] ht \rightarrow sd \in \Gamma_\delta}{[\delta] \emptyset \vdash ht \rightarrow sd}}{[\delta] \emptyset \vdash ht} \quad (2) \text{ (tr)}}{[\delta] \emptyset \vdash sd} \quad (1) \text{ (tr)}$$

SIDE CONDITIONS: (1)  $\Gamma_\pi := \Gamma_\pi \cup \{sd\}$ , (2)  $\Gamma_\delta := \Gamma_\delta \cup \{ht\}$ , (3)  $\Gamma_\pi := \Gamma_\pi \cup \{ht\}$ .

As a result of the proof construction,  $\Gamma_\pi = \{ht, sd\}$  and  $\Gamma_\delta = \{ht \rightarrow sd, ht\}$ . Note that although  $sd \notin \Gamma_\delta$ ,  $\delta$  can now infer  $sd$  without recourse to observation or communication. From the proof tree, we can read off the moves of dialogue 16. The dialogue contains a well-known conversation analytical structure, i.e., the insertion sequence (the subdialogue consisting of 3 and 4):

- (16) 1.  $\pi$ : Do I need to see a doctor?
2.  $\delta$ : Do you have a temperature?
3.  $\pi$ : Wait a minute [ $\pi$  checks her temperature], yes, I do.
4.  $\delta$ : Then you do need to see a doctor.

## 7 Related Work

In this section we contrast the approach described in this paper with other related approaches. Firstly, note that the extended Natural Deduction calculus that we employed required a number of modifications to the standard calculus employed by, for instance, Sundholm (for specifying inferential roles). We introduced a distinction between temporary and persistent assumptions and used the member rule to access both types of assumptions. Persistent assumptions were introduced to collect premises from sources other than inference (i.e., communication and observation). Another crucial extension was the explicit relativization of judgments and assumptions to agents.

Secondly, our approach differs in a number of respects from extant models of dialogue. Here we compare our approach with two representative classes of alternatives. Firstly, there is a body of work based on the idea that dialogues can be characterized in terms of information states in combination with update and generation rules. The difference with our approach is that we try to explain dialogue coherence in terms of independently motivated inferential roles of logical constants. Compare this with, for example, (Beun, 2001) who introduces special purpose generation rules to achieve the same effect as our intro and elim rules for ‘ $\rightarrow$ ’, the conversational procedures in the pioneering work by Power (1979), the up- and downdating rules for the partially ordered questions under discussion in Ginzburg (1996), and the generic framework for information state-based dialogue modelling described in Traum & Larsson (2003). For a comparison of some of these existing approaches see Pulman (1999).

Secondly, there is a dialogue game approach going back to the work of Lorenzen – see Lorenzen & Lorenz (1978) – where the logical constants are defined in terms of their role in rational debates. There the order of explanation is from (a) formal winning strategies for *adversarial* dialogues (debate) to (b) valid patterns of reasoning involving the logical constants.<sup>5</sup> In contrast, we proceed from (b) valid patterns of reasoning involving the

<sup>5</sup>Hamblin (1971) also explores derivations in this direction: *from* a specification of legal dialogue – though his dialogues are information-oriented, rather than adversarial – to semantic properties of locutions. Furthermore, the game-theoretical semantics that has been developed by Hintikka and collaborators (Saarinen, 1979) has some central features in common Lorenzen’s dialogue games.

logical constants to (c) coherent *cooperative* dialogue. The undertakings are complementary and raise the, rather surprising, prospect of an account of cooperative dialogue based on adversarial dialogue (debate); that is, an account from (a) to (c) via (b).

## 8 Limitations and Further Research

The aim of this paper is to provide the foundations for a generative logic-based model of dialogue coherence. The generic framework is described in section 4, whereas specific systems are developed in sections 3, 5 and 6. The purpose of these systems was to demonstrate that the type of analysis advocated here can account for certain dialogue structures. The systems are, however, limited in a number of ways. In this section we identify these limitations, and provide some suggestions on how to address them.

(i) Inconsistencies between participants’ informational states are avoided by the use of a minimal logic that lacks negation. In future work, we plan to add inference rules for negation, and investigate the implication of such an extension, in particular, with regards to interactions with the observation rule. (ii) We assume that dialogue participants always successfully perform speech acts: the communication channel is perfect (no misperception) and the language is fully shared and free of ambiguous expressions. (iii) Communication is mostly direct: speakers express what they mean by saying it, rather than by Gricean implicature (Grice, 1975). We intend to address implicature by extending the system with non-standard patterns of inference (e.g., default reasoning) without changing the inferential roles of the logical vocabulary. We intend to achieve this by means of accommodation rules operating on the sets of persistent assumptions; *cf.* chapter 2 of Piwek (1998). (iv) The current systems only deal with information-oriented dialogues. In future, we would like to investigate application of the current framework to task-oriented dialogues that involve imperatives and actions. (v) The current systems are based on proposition logic. We plan to develop further systems that incorporate more expressive logics, in particular, the predicate calculus. For this purpose, we will build on an implementation of a system for natural deduction for predicate and higher order logics (Piwek, 2006), and the work on consistency maintenance in type theory-based

natural deduction systems by Borghuis and Nederpelt (2000). (vi) Beun (2001) points out that his system needs to be extended with less elegant rules to prevent generation of dialogues with loops (e.g., by not allowing an agent to ask the same question twice). Our system is infected with the same problem. To avoid classifying repetitive dialogue structures as coherent, we need a rule that prevents an agent from transferring the same proof goal to the same agent more than once. (vii) Currently, our framework is set up so that proof (search) trees are produced first and then mapped to dialogue (structures). This provides us with a theoretically clean and transparent framework for relating inference systems to dialogue structure. The work also has practical potential, for example, as a framework for generating information presentations in dialogue form; see the discussion of dialogue as discourse in Piwek & Van Deemter (2002). Nevertheless, there is also scope for investigating how the mapping rules can be integrated with proof search, thus making it possible to use the resulting system in human-computer dialogue.

## 9 Conclusion

The current paper is foundational in nature. We show how to model dialogue coherence in terms of generative systems that rely on an extended calculus of Natural Deduction. At the core of this account is the standard Natural Deduction calculus which has been motivated independently. The paper presents extensions of the calculus with rules for communication and observation, and describes a mapping from proof (search) trees to dialogue structures. We hope that the current paper will stimulate discussion about the role of logic and sentence semantics in understanding dialogue coherence.

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