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Adiabatic transport of localized electrons

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Abstract. This paper discusses the effect of a low-frequency electric field on Anderson localized electrons, and a new mechanism for dissipative absorption of energy and spatial diffusion of the electrons is introduced. The mechanism is based on an application of the quantum adiabatic theorem: varying the electric field causes widely separated states to become degenerate, and if the field is varied sufficiently slowly the electron hops between the degenerate states. This effect becomes the dominant mechanism for dissipation if the electric field exceeds a threshold value, which decreases as the frequency tends to zero. The experimental observability of the effect is discussed.

1. Introduction

It is well known that electrons in a disordered solid, treated in the independent-electron approximation, can have spatially localized eigenstates, and that if the eigenstates at the Fermi energy are localized the zero-temperature DC conductivity is zero (Ishii 1973). This paper considers in detail the effect of an AC electric field on this system. This problem has previously been investigated by Mott (1970), who computed the AC conductivity $\Sigma(\omega)$ using the Kubo-Greenwood formula (Kubo 1956, Greenwood 1958), a perturbative method, and found that in d dimensions the leading-order behaviour of $\Sigma(\omega)$ is proportional to $\omega^2|\ln \omega|^{d+1}$ in the limit where the frequency $\omega \to 0$. The principal new result in this paper comes from considering a small but finite electric field, $\mathscr{E}(t) = \mathscr{E}_0 \cos(\omega t)$, and applying the quantum adiabatic theorem. This leads to a new mechanism for the transport of electrons, which becomes dominant if we consider the limit $\omega \to 0$ with the amplitude $\mathscr{E}_0$ held fixed. The paper also comments on the relationship of these results to the earlier perturbative analysis, and to other recent studies on irreversible processes resulting from non-adiabatic transitions.

Varying the electric field $\mathscr{E}$ perturbs the energy levels of the electron eigenstates, and causes a pair of states $|\phi_1\rangle$, $|\phi_2\rangle$ localized at two widely separated positions to become nearly degenerate. In the neighbourhood of the near-degeneracy the exact eigenstates $|\psi_1\rangle$, $|\psi_2\rangle$ are linear combinations of $|\phi_1\rangle$ and $|\phi_2\rangle$. If the energy levels of these two states are plotted as a function of $\mathscr{E}$ the curves do not cross: this is illustrated in figure 1. The lower eigenstate $|\psi_1\rangle$, initially very close to $|\phi_1\rangle$, is very nearly equal to $|\phi_2\rangle$ after the avoided crossing: the identity of the upper state is similarly exchanged. If $\mathscr{E}(t)$ varies sufficiently slowly, the quantum adiabatic theorem (Bohm 1951) tells us that an electron in the lower eigenstate $|\psi_1\rangle$ remains in this state, resulting in a transfer of the electron over a large distance, from state $|\phi_1\rangle$ to state $|\phi_2\rangle$. This mechanism is discussed in greater detail in the next section, and it is shown that it leads to a spatial diffusion of the electrons under the influence of a sufficiently strong AC electric field.

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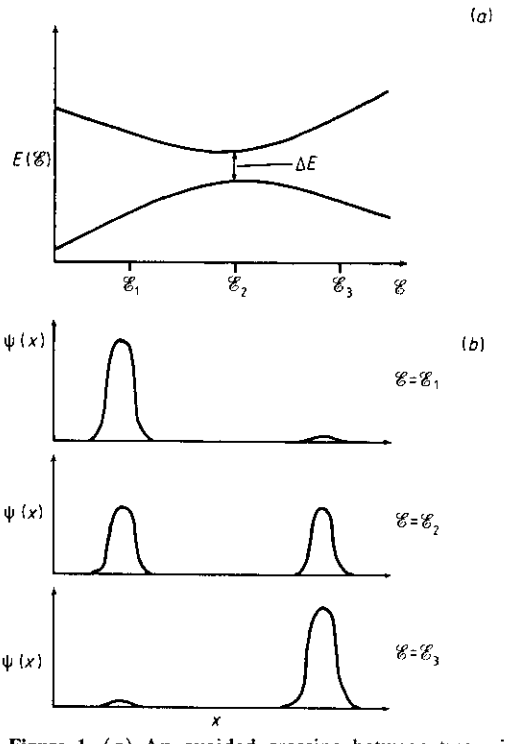

Figure 1. (*a*) An avoided crossing between two widely separated states, and (*b*) the corresponding wave function for the lower eigenstate at three values of the electric field. If the electric field $\mathscr{E}(t)$ varies sufficiently slowly, the electron is adiabatically transported to a widely separated state.

In section 3 it is shown that this adiabatic transport effect also leads to the dissipation (i.e. effectively irreversible absorption) of energy by the electron gas, and the rate of dissipation is calculated. Section 4 discusses the perturbative calculation of the conductivity, and it is argued that, although this result is formally correct, the energy absorption in the perturbative regime saturates very quickly, and that the AC conductivity may be very difficult to observe. Section 5 discusses the potential experimental observability of the adiabatic transport effect, and its relationship to some other problems involving non-adiabatic transitions and the response of systems to periodic perturbations.

2. The adiabatic transport mechanism

The model system considered in this paper is as follows. The electrons are treated as independent, spinless fermions, and the electric field as a purely classical time-dependent perturbation of the one-electron Hamiltonian, of the form $e\mathscr{E}(t)\hat{x}$. The one-electron Hamiltonian is therefore

$$\hat{H}(\mathscr{E}) = \hat{H}_0 + e\mathscr{E}(t)\hat{x} \qquad \mathscr{E}(t) = \mathscr{E}_0 \cos(\omega t). \tag{2.1}$$

The unperturbed Hamiltonian $\hat{H}_0$ is a model for a disordered solid, with Anderson localized states at the Fermi energy: as an example, $\hat{H}_0$ could be an Anderson model Hamiltonian in d dimensions with lattice constant unity, and $\hat{x}$ the index of the lattice

sites along one of the principal axes of the lattice. The eigenstates of $\hat{H}_0$ have a density of states per unit volume ρ, and a localization length ξ at the Fermi energy.

Now the adiabatic transport effect will be described in greater detail. Consider two states $|\phi_1\rangle$, $|\phi_2\rangle$, with nearly equal energies $E_1(\mathscr{E})$ and $E_2(\mathscr{E})$, which are separated by a distance L which is much greater than ξ. The effect of the perturbation can be analysed using degenerate perturbation theory: the state of the perturbed system can be approximated by the linear combination

$$|\psi\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle \tag{2.2}$$

where (a_1, a_2) is an eigenvector of the 2×2 matrix formed by the matrix elements of $\hat{H}(\mathscr{E})$ in the $|\phi_1\rangle$, $|\phi_2\rangle$ subspace. In the neighbourhood of the point $\mathscr{E}^*$ where the states are closest to degeneracy, this matrix can be approximated as follows:

$$\begin{bmatrix} E_0+(\mathscr{E}-\mathscr{E}^*)V_{11} & H_{12} \\ H_{12} & E_0+(\mathscr{E}-\mathscr{E}^*)V_{22} \end{bmatrix}. \tag{2.3}$$

The diagonal matrix elements V_{11} and V_{22} are approximately equal to ex_1 and ex_2 respectively, where x_1 and x_2 are the x-coordinates of the positions about which the states $|\phi_1\rangle$ and $|\phi_2\rangle$ are localized. The matrix element H_{12} is much smaller because of the very small overlap of the states $|\phi_1\rangle$ and $|\phi_2\rangle$. The matrix element H_{12} is of order of magnitude $I_0\exp(-L/\xi)$, where $I_0=(\rho\xi^d)^{-1}$ is the mean separation of energy levels in a block of size ξ. The energy levels are given by the eigenvalues of (2.3),

$$E(\mathscr{E}) = E_0 \pm \tfrac{1}{2}[(V_{11}-V_{22})^2(\mathscr{E}-\mathscr{E}^*)^2+4H_{12}^2]^{1/2} \tag{2.4}$$

which approach each other to within a separation $\Delta E = 2H_{12}$ at their point of closest approach, $\mathscr{E}^*$ (see figure 1). This close approach of eigenvalues is called an avoided crossing. Notice that when the parameter $\mathscr{E}$ passes through an avoided crossing, the coefficients of the eigenvector with the lowest eigenvalue change from being approximately (1, 0) to approximately (0, 1) (or vice versa). The locations in space of the lower and upper states therefore move through a distance L as we pass through an avoided crossing, and the identities of the states are exchanged. Now we recall the quantum adiabatic theorem (see Bohm 1951), which states that if we start in the nth eigenstate of the system, and the Hamiltonian is changed sufficiently slowly, then the system remains in the nth eigenstate of the instantaneous Hamiltonian. By varying the electric field $\mathscr{E}$ sufficiently slowly, we can therefore make an electron hop a distance L, which is much greater than the localization length. If the rate of change of the parameter $\mathscr{E}$ is too fast, there is a significant probability to make a transition from one state to the other: these transitions are called Landau–Zener transitions, and in the present context making the non-adiabatic transition corresponds to leaving the electron in its original location.

The probability for making a non-adiabatic transition in a simple two-level system such as that described above is given by

$$P_t = \exp(-\pi\Delta E^2/2A\hbar\dot{\mathscr{E}}) \tag{2.5}$$

where ΔE is the closest approach of the eigenvalues, A is the asymptotic difference in $\mathrm{d}E/\mathrm{d}\mathscr{E}$ for the two energy levels, and $\dot{\mathscr{E}}$ is the rate of change of the parameter $\mathscr{E}$ as it passes through the avoided crossing (Zener 1932). In order to understand the role of these non-adiabatic transitions in producing diffusion of the electrons, it is necessary to consider the statistical distribution of the parameters ΔE and A appearing in (2.5).

Consider the electronic states that would exist in a finite-sized block of material, with size $L_0 \gg \xi$. The dependence of the energy levels on the electric field $\mathscr{E}$ is shown schematically in figure 2: there are many avoided crossings, with a wide spectrum of possible gap sizes, but few of the gaps will be smaller than

$$\Delta E \simeq I_0 \exp(-L_0/\xi) \qquad I_0 = (\rho \xi^d)^{-1}. \tag{2.6}$$

Equation (2.5) shows that the effects of varying $\mathscr{E}(t)$ are reversible (i.e. the transition probability $P_t \simeq 0$), provided

$$\Delta E^2 \gg eL_0 \hbar \dot{\mathscr{E}}. \tag{2.7}$$

This estimate uses the fact that the difference A between the derivatives $\mathrm{d}E/\mathrm{d}\mathscr{E}$ for the two states is approximately eL, where e is the electron charge and L the distance between the two states. Although the electrons can move distances of order L_0, these processes are reversible and do not result in any diffusion of the electrons, provided (2.7) is satisfied.

Now consider what happens when this finite block of material is incorporated into an infinite lattice. All the states except those within a distance ξ of the edges of the block remain almost the same, except when, at a particular value of $\mathscr{E}$, they become degenerate with another state somewhere else in the lattice. Consider the crossings of a given state with the states at distances between L and $L+\delta L$. The rate at which crossings with such states occur is

$$\frac{\mathrm{d}N}{\mathrm{d}t} \simeq \rho e \dot{\mathscr{E}} L^d \, \delta L \tag{2.8}$$

where ρ is the density of states per unit volume, and the gap sizes of the avoided crossings are in the range $\delta \Delta E$ given by differentiating the relationship (2.6):

$$\delta \Delta E \simeq -\Delta E/\xi \, \delta L. \tag{2.9}$$

The rate of crossing of avoided crossings with gaps in the range ΔE to $\Delta E + \delta \Delta E$ is therefore

$$\frac{\mathrm{d}N}{\mathrm{d}t} \simeq \rho e \dot{\mathscr{E}} \xi L^d \frac{\delta \Delta E}{\Delta E}. \tag{2.10}$$

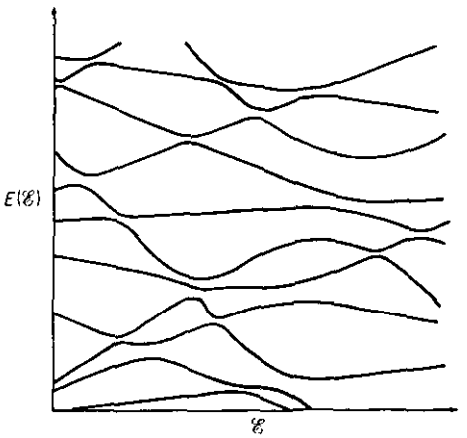

Figure 2. The energy levels of the electron states in a finite-sized sample exhibit many avoided crossings as the electric field $\mathscr{E}(t)$ varies.

Only the avoided crossings with gaps which satisfy

$$\Delta E \simeq (eL\hbar\dot{\mathscr{E}})^{1/2} \tag{2.11}$$

make any significant contribution to the diffusion of the electrons: larger gap sizes result in the adiabatic changes being perfectly reversible, and for smaller gap sizes the non-adiabatic transitions which occur do not result in any motion of the electrons. The range $\delta\Delta E$ of gap sizes contributing to the diffusive motion therefore satisfies

$$\delta\Delta E/\Delta E \simeq (1) \tag{2.12}$$

and the corresponding hopping length L is given by combining (2.11) and (2.6):

$$L \simeq \xi F(e\dot{\mathscr{E}}\hbar\xi/I_0^2) \tag{2.13}$$

where $F(x)$ is a solution of the equation

$$F = -\ln(xF). \tag{2.14}$$

Note that the function $F(x)$ is similar to $-\ln(x)$.

The diffusion constant for the electrons is

$$D \simeq \frac{dN}{dt} L^2 \tag{2.15}$$

where L is the dominant hopping distance, given by (2.13). Substituting (2.12), (2.13) into (2.10) and (2.15), gives the following estimate for the diffusion constant:

$$D = K_1 \rho e\dot{\mathscr{E}}_0 \xi^{d+3} [F(e\dot{\mathscr{E}}\xi\hbar/I_0^2]^{d+2} \tag{2.16}$$

in the limit $\omega \to 0$, where K_1 is a dimensionless constant. This result is valid provided the amplitude of the electric field is sufficiently large that the number of avoided crossings which contribute to the diffusion is large. The number of participating avoided crossings can be estimated from (2.10) and (2.12):

$$N \simeq \rho e\mathscr{E}_0 \xi^{d+1} [F(e\mathscr{E}_0\hbar\omega\xi/I_0^2)]^d \tag{2.17}$$

where we have used the estimate $\dot{\mathscr{E}} \simeq \mathscr{E}_0\omega$. The threshold amplitude $\mathscr{E}_0$ required to satisfy the condition $N \gg 1$ therefore decreases as $\omega \to 0$ because F increases in this limit.

The diffusion of electrons due to the adiabatic transport effect could be observed by detecting a zero-temperature DC conductivity induced by the application of an AC electric field. The DC conductivity Σ_0 can be calculated from the spatial diffusion constant D using an equation known as the 'zero-temperature Einstein relation',

$$\Sigma_0 = e^2 \rho D \tag{2.18}$$

which is derived in the same way as the usual Einstein relation.

3. Relation between diffusion and dissipation

In the previous section it was shown that the oscillating electric field causes a spatial diffusion of the electrons. There is a corresponding diffusion of the energy of the electrons, with diffusion constant D_E: the change in energy of an electron satisfies

$$\langle \Delta E^2 \rangle = 2D_E t \tag{3.1}$$

where ΔE is the change in the electron energy in time t, and the angular brackets denote an average over different electrons. This section estimates the diffusion constant D_E and shows how this quantity is related to the dissipation of energy.

The diffusion constant D_E can be estimated very simply: irreversibility in the dynamics is introduced by avoided crossings at which the transition probability is neither very close to unity or to zero. The frequency dN/dt of these avoided crossings is given by (2.10) and (2.12) above. Between these avoided crossings the energy of the electron changes by an amount of the order of $\delta E \simeq eL\dot{\mathscr{E}}\delta t$, where $\delta t = (dN/dt)^{-1}$ is the typical time interval between these avoided crossings. Modelling the dynamics, in energy space, of the electron as a random walk with steps of size δE occurring at a rate $dN/dt \simeq 1/\delta t$, leads to the estimate

$$D_E \simeq (\delta E)^2 \, dN/dt. \tag{3.2}$$

Substituting for dN/dt using (2.10), (2.6) and (2.12):

$$D_E = \frac{K_2 e\dot{\mathscr{E}}}{\rho \xi^{d-1}} [F(e\dot{\mathscr{E}}\xi\hbar/I_0^2)]^{2-d} \tag{3.3}$$

where K_2 is a dimensionless constant.

Now consider how this diffusion constant relates to the dissipation of energy. Assume that the system containing N_F electrons is initially in the ground state, with all the states up to N_F filled, and all the others empty. At a later time t, the total energy of the system is

$$E_T(t) = \sum_{i=1}^{N_F} \sum_j |U_{ji}(t)|^2 E_j(t) \tag{3.4}$$

where the $U_{ij}(t)$ are matrix elements of the evolution operator in the adiabatic basis (i.e. $U_{ij}(t)$ is the amplitude to be in the ith state of the instantaneous Hamiltonian at time t, having started in the jth state of the Hamiltonian at time 0). An elementary calculation shows that, if the summations are approximated by integrals, the change in the total energy of the system at time t is

$$\Delta E_T(t) = \frac{1}{2\rho V} \int_{-\infty}^{\infty} dn \, n^2 P(n, t) \tag{3.5}$$

(Wilkinson and Austin 1990, 1991) where V is the volume of the system, and $P(n, t)$ is the average probability to make a transition from the ith state to the $i+n$th state,

$$P(n, t) = \langle |U_{i+n,i}(t)|^2 \rangle_i \tag{3.6}$$

with the average confined to states near the Fermi energy, $i \simeq N_F$. If the transitions are diffusive, as described above, the distribution $P(n, t)$ is Gaussian, and

$$\int_{-\infty}^{\infty} dn \, n^2 P(n, t) = 2\rho^2 V^2 D_E t \tag{3.7}$$

so that the rate of dissipation is given by

$$\frac{d\Delta E_T}{dt} = \rho V D_E \tag{3.8}$$

where D_E is given by (3.3). This result is valid provided the number of avoided crossings, given by (2.17), satisfies $N \gg 1$ so that diffusion in energy does occur.

4. Perturbative theory for the rate of dissipation

In this section the AC conductivity is derived via a calculation of the rate of dissipation: this result was originally obtained by Mott (1970) using the Kubo-Greenwood formula. It will be argued that the total energy which can be absorbed by this mechanism is very small, making an experimental observation of the AC conductivity difficult.

The conductivity, $\Sigma(\omega)$, is simply related to the mean rate of dissipation $\mathrm{d}\Delta E_T/\mathrm{d}t$:

$$\left\langle \frac{\mathrm{d}\Delta E_T}{\mathrm{d}t} \right\rangle = \tfrac{1}{2} V \Sigma(\omega) \mathscr{E}_0^2 \tag{4.1}$$

where the brackets denote a time average, and V is the volume of the sample. Note that the conductivity is only defined if the rate of dissipation is proportional to the square of the electric field, and that it is therefore not possible to characterize the rate of dissipation obtained in the previous section by a conductivity.

The probability for making a transition through n states, $P(n, t)$, is calculated using perturbation theory: after $N \gg 1$ cycles of the applied electric field, only states separated by $\pm\hbar\omega$ in energy have a significant transition probability:

$$P(n, t) = (1 - 2Rt)\delta(n) + Rt\delta(n + \rho V \hbar\omega) + Rt\delta(n - \rho V \hbar\omega). \tag{4.2}$$

This result remains valid for times short enough that $Rt \ll 1$. The transition rate R is given by a version of the Fermi golden rule:

$$R = \frac{\pi}{2\hbar} \rho V \sigma^2 \mathscr{E}_0^2 \tag{4.3}$$

where σ is the RMS matrix element coupling states near the Fermi energy which differ in energy by $\hbar\omega$,

$$\sigma^2 = \frac{1}{V^2 \rho^2} \sum_n \sum_{m \neq n} \left(\frac{\partial \hat{H}}{\partial \mathscr{E}} \right)_{nm} \delta_\varepsilon(E_n - E_F) \delta_{\varepsilon'}(E_n - E_m - \hbar\omega). \tag{4.4}$$

In (4.4) $\delta_\varepsilon(x)$ is a pseudo-delta function of width ε, and the matrix elements are those of the dipole operator $\partial \hat{H}/\partial \mathscr{E} = e\hat{x}$ in the basis formed by the eigenstates of $\hat{H}$.

Mott (1970) pointed out that the average (4.4) is dominated by matrix elements coupling pairs of states which are in resonance: two states $|\phi_1\rangle$ and $|\phi_2\rangle$ separated by a distance L, which is large compared to the localization length ξ, have nearly identical energies, and the exact eigenstates are linear combinations of these two states. A pair of resonant states, occurring at electric field $\mathscr{E} = \mathscr{E}_2$, is illustrated in figure 1. The matrix elements of $\hat{x}$ are clearly of order of magnitude L for these resonant states. The matrix elements of $\hat{x}$ for pairs of states with similar energies are of order of magnitude ξ for pairs of states centred within a localization length of each other, and are exponentially small, of order of magnitude $\xi \exp(-L/\xi)$, for other pairs of states which are not in resonance.

Now consider the problem of counting the set of states which are in resonance with a given state, and which provide the dominant contribution to (4.4). For a pair of states to be in resonance the matrix element $\Delta E/2$ splitting the degeneracy, given by (2.6), must be approximately equal to $\hbar\omega$,

$$\Delta E/\hbar\omega \simeq 1 \tag{4.5}$$

or equivalently ΔE can differ from $\hbar\omega$ by a tolerance $\delta\Delta E$ which satisfies $\delta\Delta E/\Delta E = O(1)$. The range of separations δL over which the resonance occur is given by differentiating (2.6),

$$\delta L \simeq -\xi\,\delta\Delta E/\Delta E \simeq \xi \tag{4.6}$$

so that the resonant states are contained in a shell of volume δV:

$$\delta V \simeq L^{d-1}\xi. \tag{4.7}$$

The average (4.4) can now be evaluated:

$$\sigma^2 \simeq \frac{e^2\delta V}{V}L^2 \simeq e^2\xi^{d+2}V^{-1}\left[\ln\left(\frac{\hbar\omega}{I_0}\right)\right]^{d+1}. \tag{4.8}$$

Substituting (4.8) into (4.3), using (4.2) to evaluate the integral in (3.5), and comparing with (4.1), the AC conductivity is therefore

$$\Sigma(\omega) = K_3\frac{e^2}{\hbar}(\hbar\omega)^2\rho^2\xi^{d+2}\left[\ln\left(\frac{\hbar\omega}{I_0}\right)\right]^{d+1} \tag{4.9}$$

where K_3 is a dimensionless constant, which is the result obtained by Mott (1970). This result is valid for low frequencies satisfying $\hbar\omega \ll I_0$. Numerical results verifying that this result is a correct evaluation of the Kubo-Greenwood formula in the limit $\omega \to 0$ have been described by Thouless and Kirkpatrick (1981).

Although this result is formally correct, it is questionable whether it could easily be observed experimentally. The reason for this is that most states are resonantly coupled to at most one other state, so that, in an approximation which considers only the resonant couplings, the system behaves as a collection of essentially independent two-state systems. Each of these two-level systems initially absorbs energy rapidly from the perturbing electric field, but then undergoes Rabi oscillations between the upper and lower states. The Kubo-Greenwood formula therefore correctly predicts a rapid initial absorption of energy by the system due to the resonances, but ignores the fact that this mechanism of absorption saturates, with only a few electrons having absorbed at most one quantum, $\hbar\omega$, of energy.

The same comment could equally well be made about the application of the Kubo formula in other circumstances: it is essentially just a computation of resonant absorption using the Fermi golden rule. In most applications of the Kubo formula, however, each state is coupled to many others by matrix elements of comparable magnitude. In these cases small perturbations due to phonons or electron-electron interactions can destroy the phase coherence required for the resonant absorption to saturate, and enable further excitation of the system. In the case considered above, where there is strong coupling to at most one other state, there is no possibility for further excitation of the system once the upper state is saturated. The weaker couplings to states within a localization length would still enable some further absorption of energy, but at a much lower rate.

It is easy to verify that the average number of states in resonance with a given state is small using the estimates given above. The width ε' of the delta function of $\Delta E - \hbar\omega$ in (4.4) must satisfy $\varepsilon' \ll \hbar\omega$. The average number of states in resonance with a given state, δN, is therefore

$$\delta N \simeq \rho\delta V\varepsilon' \ll \rho\hbar\omega\xi^d\left[\ln\left(\frac{\hbar\omega}{I_0}\right)\right]^{d-1} \ll 1 \tag{4.10}$$

where the final inequality results from the fact that we are considering frequencies small enough that $\hbar\omega$ is small compared to the local mean level spacing, $I_0 = (\rho\xi^d)^{-1}$.

5. Discussion

This paper has described a novel mechanism for spatial diffusion of electrons and dissipation of energy induced by a finite-amplitude AC field. Comparing the rate of dissipation due to this mechanism with that computed from the perturbative expression for the AC conductivity shows that the adiabatic transport mechanism is dominant in the limit $\omega \to 0$ with the amplitude $\mathscr{E}_0$ held fixed. The adiabatic transport mechanism only applies if the number N of avoided crossings is large, but note that $N \to \infty$ as $\omega \to 0$ with $\mathscr{E}_0$ fixed (see (2.17)). Also, it has been argued that the rate of dissipation in the perturbative case is lower than the estimate obtained from (4.1) and (4.9), because the energy absorbed by excitation of resonant states soon saturates.

Although the adiabatic transport effect is non-perturbative, the conditions required to reach the threshold intensity are not extreme. Consider a d-dimensional system with localization length $\xi = \alpha\lambda_F$, where λ_F is the Fermi wavelength, and Fermi energy E_F. The density of states is $\rho \simeq (E_F\lambda_F)^{-1}$, and the local mean level spacing is $I_0 \simeq E_F/\alpha^d$. ignoring the factor F^d in (2.17), the number of avoided crossings is approximately $N \simeq \alpha^{d+1} e\mathscr{E}_0\lambda_F/E_F$. The frequency ω^* below which adiabatic effects become important is given by equating $\hbar\omega^*$ to the local mean level spacing, I_0. Taking some typical values for experiments on a two-dimensional electron gas, $\lambda_F = 100$ Å, $E_F = 10$ meV, $\mathscr{E}_0 = 10^4\ \mathrm{Vm}^{-1}$, and assuming $\alpha = 100$, gives $N \simeq 10^4$, with $\omega^*/2\pi = 0.25$ GHz. Thus a large number of avoided crossings can be caused by the application of a modest electric field, if the localization length is large compared to the atomic length scale.

It has been observed that, in a very wide class of systems in which the Hamiltonian is a periodic function of time, the energy transferred by a time-periodic perturbation saturates. This is due to Anderson localization of the eigenstates of the Floquet operator, expressed in the adiabatic basis: the effect is termed 'localization in energy' (see Wilkinson and Austin 1990, and references therein). The system considered here is certainly expected to exhibit energy localization, but in most regimes of the model it would only affect the dynamics after a very large number of cycles: the exception to this is the low-frequency perturbation discussed in section 4. Some results which can be used to estimate the localization properties will be described in a future publication (Wilkinson and Austin 1991).

A related mechanism for dissipation involving Landau–Zener transitions has also been described in finite-sized systems, in which the statistical properties of the energy levels can be described by random matrix models (Wilkinson 1988, 1990). This analysis leads to a different dependence on frequency from the case considered here.

Phonons could also provide the necessary perturbation to cause diffusion of electrons by the adiabatic transport effect: this is a new mechanism for phonon-induced hopping conductivity, and further work is being done to determine the circumstances under which such a mechanism could be significant.

6. Acknowledgments

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