# Structural Variation Grids

### Historical Context

Many years ago I experienced a lesson given by Laurinda Brown based on the function game (Banwell *et al* 1972, see also Rubenstein 2002). It was conducted entirely in silence to great effect. Participants were invited to conjecture the result of applying an unknown function to different inputs, based on examples provided by her at the beginning. Everything was done in silence, with sad or happy faces drawn according to whether the *keeper of the rule* agreed or disagreed with the conjecture. The one rule was that no-one was allowed to say what they thought the rule was. Those who thought they knew ‘the rule’ were encouraged to offer examples which would help others come to the same conjecture, and also to try to test and challenge their conjecture. Apart from the silence, the format has strong resonances with the game *Eleusis* described by Martin Gardner (1977; 2001 p504–512). Gardner observes that the rules provide an analogy with science, because nature never tells you whether your conjectured rule is correct.

I was stimulated to look for the first opportunity to try working in silence and it came in a lecture to 300 Open University students, in which I presented the first few terms of a sequence:

2 + 2 = 2 x 2   .

I paused at each equal sign, and at the end of each equation, in order to show that I was doing the calculations myself. I have since done this with thousands of people over many years. Each time, no matter who the people are, everyone seems to know what the next term will be even if they struggle with the arithmetic to check the validity of their conjecture. I have used many sequences like this, getting participants to re-present the first term in the format of the others, to go backwards into the negatives (starting with 0, then -1, -2, …), to use not just whole numbers but rationals (starting with ½ or ¾), irrationals (starting with or ) and beyond, according to the sophistication of the audience. The main thrust is towards expressing the general equation, and then justifying it using algebra. Sequences like this can be used to provoke learners into wanting a way to manipulate generalities (letters), as well as a providing a source for appropriate rules for that manipulation: the rules of algebra as generalizations of the rules of arithmetic. This contrasts with algebra presented simply as rules for ‘alphabet arithmetic’.

In 1998 I was asked by some teachers in Tunja Colombia to suggest how to work with learners on factoring when they did not have facility with or even belief that (-1) x (-1) = 1. My response was what I then called *Tunja Sequences* (Mason 1999, 2001) which used the same principle of a developing sequence of specific instances of a factored quadratic such as

12 – 12 = (1 – 1)(1 + 1) 22 – 1 = (2 – 1)(2 + 1) 32 – 1 = (3 – 1)(3 + 1) …

Here learners could be expected to detect the pattern and to express it in general, verbally, and even algebraically. By being exposed to a number of such sequences derived from factored quadratics, learners could be expected to become adept at expressing and justifying generality (the heart, root and purpose of algebra). Having generalised, they can work out the rules for expanding brackets, and for factoring quadratics, simply by using their natural powers to detect what is changing and what is invariant.

Recently, while writing a book on the teaching of algebra (Mason *et al* 2005) I wanted to extend these Tunja sequences to allow a second parameter to vary, and thus was born Structural Variation Grids. Tom Button kindly provided me with a basic Flash template which I then modified to produce different Grids, some of which are described in the next section.

Using these grids briefly with teachers has already generated considerable excitement, and this is what has encouraged me to present them in this forum. I am confident that many of you will have done or used something similar at various times. The reason for presenting them here is to exhibit them as an exemplar of a pedagogic framework for learning.