Responsive & Responsible Teaching:

so, what is your theory?

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Mathematics Education Research and Mathematics Teaching:   
Illusions, Reality, and Opportunities

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He who loves practice without theory is like the sailor   
who boards ship without a rudder and compass   
and never knows where he may cast.   
Practice always rests on good theory.   
(Leonardo Da Vinci)

It is only after you come to know the surface of things  
that you venture to see what is underneath;  
But the surface of things is inexhaustible  
(Italo Calvino: Mr. Palomar)

## Abstract

Responsive teaching is responsive to learners’ states and actions and to the integrity of the mathematical thinking behind curriculum topics. Responsible teaching means being able to justify your choices of actions both when preparing and in the minute-by-minute flow of the classroom. In order to teach responsibly it is necessary to be articulate about what informs choices. Attempts to become articulate about reasons both reveal and develop personal theory.

Questions:

What research informs mathematics teaching?

How does this research come to be put into practice?

What are the issues and gaps?

Conjecture: research that can usefully inform curriculum and task design and classroom practice is multi-layered, foregrounding awarenesses, sensitivities and actions to initiate and carry through, showing how they align with basic assumptions and stances, how they played out in particular instances, but without trying to claim that this or that ‘works’.

Conjecture: what matters most is the *Being* of the teacher in relation to both learners and mathematics.

## Background

In the early 1970’s, when I had been working at the Open University for a few years writing distance-learning mathematics materials, I was asked “so, what is your theory?” by some German visitors. I was dismayed that I did not have a coherent response, and resolved to do something about it. Over the years I have accumulated a large collection of slogans and technical terms which serve to remind me of choices that can be made in situations where those choices actually come to mind. The slogans link to the technical terms, many of which are drawn from the research literature. I have tried to become articulate about the reasons and expectations for making various pedagogic and mathematical choices. My attempts were certainly amplified by the fact that I was employed to write courses in mathematics and mathematics education as professional development for teachers.

I want to distinguish (though not sharply) between *reactive*, *responsive, reflective* and *responsible* teaching, in order to urge myself to strive to be responsible. *Reactive teaching* consists of actions which are initiated intuitively as they come to mind in reaction to learner engagement in the flow of interaction. Typically, lessons are planned and enacted with attention focused on institutional curricular demands rather than on learner behaviour. *Responsive teaching* involves thoughtful, even principled response to learner behaviour, but the teacher may not have recourse to articulate justification of actions and choices. *Reflective teaching* includes specific and intentional reflection on actions, where what has happened recently is re-considered and used to inform future action, with or without articulate justification. *Responsible* *teaching* is making choices to act for which expectations can be articulated, and which can, if required, be justified through use of technical terms, assumptions and values. In between there is *reflective teaching*, in which what has happened recently is re-considered and used to inform future action, with or without articulate justification.

I propose that as mathematics educators we have a responsibility to promote *responsible teaching* there is growing evidence that particular teaching practices are effective only when teachers understand the principles that underpin them: Beswick (2013), quoting Askew, Brown, Rhodes, Johnson & William (1997), Watson & de Geest (2005), and Beswick (2007)

How does one justify choices of actions? I suggest that it is by recourse to *theory*, that is, to technical terms, values and assumptions concerning the lived experience of learners of mathematics. Many authors, too numerous to mention, equate assumptions with beliefs. However beliefs are for me an observer’s construction in order to explain or account for observed behaviour. Where assumptions, being made explicit, can be challenged, leading to modification, beliefs are usually seen as having a more rigid structure.

## What is Your Theory?

In mathematics, the notion of a *theory* is clear: a set of axioms together with all the deductions that can be made from those axioms, preferably with at least one model so that the theory is known not to be vacuous. The importance of a theory is that whenever the axioms are instantiated, the theorems provide consequences that must be true in any situation in which the axioms hold.

One of the non-mathematical aspects of mathematics education is that there are no sets of axioms from which deductions can be made, only stances which align with some practices but not others. In mathematics education, the notion of a *theory* is not at all clear. What passes for theory in most research journals is a framework of distinctions used to label phenomena discerned in situations. These may be explicit before data is collected, or may emerge from discerning distinctions in the data. Frameworks are lenses, through which to observe. Observations are made using those distinctions, and this passes for analysis. It is often unclear whether the framework is being illustrated by the data, or is being revealed through acting upon on the data. The most effective distinctions are those that are linked to non-habitual actions so that when a distinction is noticed, a fresh possibility of action comes to mind before habit kicks in.

For example, in the midst of a lesson, you notice an opportunity to pause and invite learners to express for themselves, in their own language, something that has been said or done, discovered or brought back to mind. This might be simply for themselves, or as a topic of discussion with one or more colleagues. What gives the noticing strength may be a commitment to mathematics and to the importance of learners recognising and developing their power to express generality. Or it may be a commitment to learners, to provide space, time and practices for making personal sense of the flow of ideas and possibilities, perhaps by summarising, making a précis (so as to be more precise) or by rehearsing.

There are certainly background theories, that is, assumptions about how learning progresses. Some have attributes of *staircase* theories, others of *spiral* theories, and others of *maturation* theories.

In *staircase* theories of learning there is an implicit or explicit assumption that learning takes place as a series of levels, steps, or advances. The UK National Curriculum (1990-2013) was constructed as a series of levels and sublevels forming a staircase of progression. Mastery Learning (Bloom 1971), Van Hiele theory of geometrical learning (van Hiele-Geldof 1957) and more generally (van Hiele 1986) are explicit examples. When learners are shown worked examples, invited to ‘do thou likewise’ (Cardano 1545/1969, Gillings 1972), and to practice regularly as they ascend the staircase of competence, the observer detects a ‘staircase theory’ of learning.

In *spiral* theories of learning there is an implicit or explicit assumption that learning deepens with frequent returns to the same ideas[[1]](#footnote-1). Bruner (1966) and the Pirie-Kieren (1989) onion model of the growth of understanding are explicit examples, and the MGA spiral (Manipulating; Getting-a-sense-of; Articulating) was one of several spiral frameworks derived from Bruner and used with teachers (Floyd *et al*. 1985) to good effect.

In *maturation* theories, learning is seen as involving something akin to phase-transitions which take place over time, somewhat akin to the formation of beer, wine or bread. Here the emphasis is on immersion in practices, without the expectation that learners will quickly display changed behaviour. While “teaching takes place *in* time, learning is seen as taking place *over* time” (Griffin 1989). Young children learning the past tenses of irregular verbs is one well studied phenomenon that fits this description, as does the discovery some years after an examination that you actually know more than you thought you did. Brenda Denvir and Margaret Brown (1986) found that low attaining learners sometimes displayed improved performance on aspects of mathematics unrelated to the topic of the actual lesson.

Even in such well established stances there is little resembling a mathematical theory. They are much more like assumptions which guide the choice of actions, ways of seeing (the root meaning of *theory*) or perceiving. At best one can say that where it is detected that learning did not take place, there was some weakness or absence in some step along the way. For example, according to the theory of *Structure of Attention* (Mason 1998), there are several distinct states or ways of attending to something, and that if teacher and learners are attending differently, even if they are attending to the same thing, then communication between them will be at best impoverished and at worst will break down altogether. This is a *negative theory*, or a *negative use of theory*: it predicts impoverished learning under certain circumstances.

The pragmatics of education mean that policy makers, leaders and teachers would like to know ‘what works’ and what does not. But all attempts to turn theoretical frameworks into recipes for action will founder. The reason is that human beings are agentive organisms not machines, and that trained behaviour may only ‘work’ in local conditions. It is certainly true that focussing on mechanical aspects of human behaviour (habits and propensities) can show short-lived success through training. But as soon as conditions change, trained behaviour becomes useless without educated awareness to guide it. This applies both to teachers and to learners, which is why ‘drill and practice’ can get learners through tests and even a few examinations, but leaves them feeling they don’t understand, and vulnerable to changed conditions.

Most theoretical frameworks eventually get turned into behaviours. For example, one of the early responses to Piaget was an attempt to accelerate learners’ progress through the various stages delineated by the translators of his work. Vygotsky’s *zone of proximal development* was proposed as a description of how development takes place, namely, drawing attention to a phase or state in which learners are on the edge of being able to initiate for themselves, actions which currently they can carry out when cued. This is in alignment with a view of learning as a series of phase-transition in which they appear to ignore prompts and instructions and then suddenly internalise them and move on. However the ZPD was quickly turned into a zone in which teachers might act. It was used to describe what teachers were and could be doing, all based on Vygotsky’s one quoted instance. Discovery learning became first a catch-phrase and then an object of criticism and derision by reductionist interpreters who tried to turn it into a set of practices.

Nowhere has this been more marked than with the various constructivisms that have been incorporated into and then rejected by the main stream of mathematics education. With roots in the thinking of Giambini Vico and St. Augustine, and articulated by Jean Piaget as *genetic epistemology*, the learner is seen as an active meaning-maker. This turned into *naïve psychological constructivism* (learners construct meaning so don’t tell them things) which was then contrasted with *radical constructivism* (knowledge is a human construction not a description of some external world; there are close alignments with some Buddhist tenets and with *enactivism*) and replaced by *social constructivism* (behaviour and meaning are enculturated by participation in social practices) with offshoots such as Sfard’s ‘commognition’ (mathematics resides in acts of communication and so is discourse). People have tried to convert one or other constructivist perspective into a set of practices, seduced by the shift of meaning when the noun constructivism is turned into the adjective ‘constructivist’ and used in such meaningless manners as ‘constructivist classroom’, ‘constructivist teacher’ and ‘constructivist curriculum’.

Sriraman & English (2010) collected descriptions of a range of what purport to be theories used in mathematics education, with commentaries and critiques. It goes into much more detail than can even be hinted at here, but with to my reading, mainly the same conclusions: the role of theory in mathematics education is perspectival: assumptive and descriptive rather than consequential. Interestingly there is little or no mention of phenomenological perspectives, in which the focus is on lived experience. Yet in order for research to inform practice, it must influence and enrich lived experience.

## Question 1: Research Informing Practice

Conjecture: research that can usefully inform curriculum and task design and classroom practice is multi-layered, foregrounding awarenesses, sensitivities and actions to initiate and carry through, showing how they align with basic assumptions and stances, how they played out in particular instances, but without trying to claim that this or that ‘works’.

Research tends to inform the researcher more than anyone else, for in researching, the researcher learns about themselves, their sensitivities to notice and discern. It is about relationships they recognise, and generalities they detect as being instantiated. The reader of research reports has to take into account that they are learning about the researcher as much as about the researched. What informs the reader of research are previously-unmade distinctions and possible actions that come to mind as a result of reading or otherwise encountering the research. Resonance and dissonance with current sensitivities can open up to scrutiny assumptions and choices of actions as domains of personal enquiry. If research is read as deciding between different actions, then great care is needed to discern the significant conditions likely to be of significance. For example failure to do this considerably mars some attempts at international comparisons.

If mathematics education had positive theorems (under these conditions, this must be the case), then the research literature could be scoured for consequences, for actions which ‘work’. Although this is what policy makers want, mathematics education is not about machines (mechanical; repetitive) but rather about human beings (potentially agentive; potentially rational). Cause-and-effect is not a dominant mechanism when human beings are involved. There are no practices that ‘work’ independently of the context and conditions, and these cannot be specified sufficiently precisely, or perhaps even enumerated, so as to guarantee results.

Instead, there is plenty of research about topics that learners do not readily master, topics that teachers do not readily master, and practices that teachers do not readily enact. All too often this is rephrased as ‘topics that learners find difficult’ or ‘topics that learners struggle with’, but these are formulations of an observer. The teacher may struggle to get learners to perform as expected on tests, but that is not the same thing as learners struggling. Learners do undoubtedly struggle, but their lived experience is not that of the teacher.

It is refreshing to have waves of novel ideas wash through mathematics education, bringing fresh distinctions from other disciplines; it is depressing to see how often fresh distinctions are taken up as if they were the ‘silver bullet’ that could transform the lived experience of learners of mathematics.

A current example is the notion of *transformative learning*. An immediate reaction might be: is there anything one would want to call ‘learning’ that is not transformative? Promoters are trying to draw attention to significant learning beyond that accumulation of a few facts and procedures. Whatever it signifies, *transformative learning* is non-linear, non-causal, non-sequential and un-predictable with any degree of certainty (Lange 2012 p205). One way to promote such learning is through the use of narrative, getting learners to ‘construct their own story’ about concepts and about procedures. The role of narrative in learning was promoted by Bruner (1990, 1991) but it is present in Plato’s Socratic dialogues, and it is currently receiving fresh and focused attention in an attempt to turn exploit the social dimension of mathematics classrooms. These range from George Pólya’s phase of *looking back*, through *reflection* *on* action in order to promote *reflection in* action (Schön 1983), and which is the aim of the *Discipline of Noticing* (Mason 2002), to general education papers with titles such as *Transformation as Embodied Narrative* Clark (2012) and *Storytelling and Transformative Learning* (Tyler and Swartz 2012). To my mind, what would be useful would be a collection of practices, of pedagogic strategies (and perhaps some topic-specific didactic tactics) which are mathematics specific.

Other trends to be on the look out for are *threshold concepts* (concepts which if not internalised and appreciated will block further development) and *multiple selves* (the notion that each person’s psyche is comprised of multiple personalities or selves with idiosyncratic propensities, dispositions, habits and ways of channelling energies).

What to me is valuable is research that reveals different ways in which learners re-construct procedures from fragments of other procedures (for example Brown & van Lehn 1980, van Lehn 1989); different ways that learner attention can be induced to shift towards mathematically significant ways of attending; different awarenesses that comprise a rich appreciation of concepts and associated procedures; different ways of helping learners harness their energies and be induced to encounter challenges they would not otherwise have considered.

## Question 2: How does research come to be put into practice?

Conjecture: what matters most is the being of the teacher in relation to both learners and mathematics.

Few people are convinced to act against their current assumptions-values-habits (their better judgements) by research claims that some specified action is effective. There must be a mixture of dissatisfaction with the current situation and some alignment between proposed actions and underlying assumptions-values-habits. Trying to change other people almost always activates its opposite, eventually, amplifying pendulum-like swings.

Effective mediation between research and practice involves the design of tasks together with pedagogic practices and didactic tactics which provide participants with a taste of the distinctions being offered, and an awareness of relevant practices. Sometimes this will mean challenging assumptions, predispositions and perspectives, but within a supportive and sustained environment so that changes in narrative go hand in hand with changes in practices and changes in perspective. Mathematical tasks that can be used with learners, need to be accompanied by emphasis both on the affordances of the tasks and on effective ways of working with them, in order to counteract the notion that ‘rich tasks’ will, of and by themselves, cause learning.

A successful CPD session is one in which participants can imagine themselves acting differently in the near future. Support for trying out that action (it is not always easy to remember in-the-moment) and for refining and modifying it to meet current conditions is much more effective than training programmes trying to alter people’s behaviours radically. The *Discipline of Noticing* (Mason 2002) is one articulation of how practice can be developed, through each individual developing and enriching their sensitivity to their own lived experience and that of learners.

Holding the conjecture (believing) that all learners can succeed, can use and develop their natural powers, is essential if teachers are to be effective in inspiring their learners to respond positively to mathematical challenge. I see teaching as a caring profession. Teaching mathematics is a bi-caring profession: caring passionately for learners and caring passionately for mathematics.

It is all too easy in mathematics education research to lose touch with the core aim and intention, namely to improve the lived experience of learners of mathematics. Changing one’s practices as a teacher, or worse, trying to change other people’s practices, misses the point. What matters is what the learner experiences. Mathematics education has a long history of looking for simple cause-and-effect, for ‘silver bullets’ that will have immediate and significant impact on learner performance on tests, as can be seen by perusing the titles of articles in journals over an extended period of time. But mathematics education is a caring profession. It involves human beings interacting with the whole of their psyche (cognition, enaction, affect and will-attention). It is about relationships between people and relationships with mathematics. For example, (Handa 2011) gives a graphic description of becoming aware of and sensitised to the vital importance of relationship, both with mathematics and with learners, in order to be and effective teacher.

## Question 3: Issues & Gaps

The institutions through whose aegis teaching takes place are not currently conducive to or supportive of a career-long process of personal and professional development. Metaphors of control predominate, ignoring more apposite metaphors derived from ecology. Absence of trust and respect filters down through the institutions of government and education and are played out in classrooms, creating in many cases social and psychological barriers to effective learning. Scholarship, indeed professionalism, is not valued sufficiently. In the UK, a proposal to create a ‘royal college of teaching’ (to replace the politically discredited ‘teaching council’) is revealing a culturally perceived parallel with the royal college of nursing, rather than with the royal college of surgeons: teachers doing what leaders direct.

As already mentioned, there is a great need for effective mediators between research and practice. There is also a decided need for people who can mediate between the pragmatics and impoverished discourse of ‘seeking what works’ so as to formulate educational policy (control) and the discourse of researchers whose main findings are that more research is required, because focused enquiry raises more questions than it answers.

One of the roles of mathematics educators must be to challenge the discourse of policy formulation. As soon as a term is adopted by policy makers, alternative discourse which questions underlying assumptions made by the previous discourse needs to be formulated, before its associated practices become statutory. Flux and change is the essence of any organic metaphor and needs to be respected and celebrated, not subverted. The underlying metaphor of education as a factory, based on the mechanism of simple cause-and-effect needs to be challenged at every level. Maintaining complexity, respecting human beings as agentive, desirous and value-directed, and respecting mathematics as a mode of enquiry and world-perspective requires on-going elaboration and support.

## Summary

Some suggestions:

### What research informs mathematics teaching?

Research that resonates or creates constructive dissonance with prior experience, sensitising practitioners to distinctions that are associated with preparation or classroom actions, whether pedagogic strategies or didactic tactics.

### How does this research come to be put into practice?

Through experiencing freshly something of the possible lived experience of learners, by engaging in relevant mathematical tasks and reflecting on the awarenesses and the associated pedagogic strategies and didactic tactics.

### What are the issues and gaps?

Mediation between policy makers (and other leaders) and researchers.

Mediation between research findings (frameworks of distinctions with associated actions) and practice (planning and teaching).

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1. We shall not cease from exploration  
   And the end of all our exploring  
   Will be to arrive where we started  
   And know the place for the first time. (T. S Eliot: Little Gidding V L 26-29) [↑](#footnote-ref-1)