Some Mathematical Tasks  
with  
Pedagogical Purpose

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### Out of School Contexts

###### Likelihood

The results of a medical test show that 10% of the population display a particular characteristic, but 40% of the people developing a particular syndrome also display that characteristic. This means that people displaying the characteristic are 6 times as likely to develop the syndrome as people not displaying the characteristic. Generalise.

###### Currency Exchange

At the airport you see rates for exchange to another currency:

Sell: for £1 you get €1.224; Buy: for €0.817 you get £1.

Assuming the same percentage fee for each transaction, what are they charging you for conversion?

If the actual quoted exchange was £1 = €1.2425 or €1 = £0.8048, what are the transaction fees?

###### Leakage

Interviewed on the radio about leakages, a spokesperson said “We lose 40% of our water through leaks so we need to make our reservoirs 40% bigger”. What?

###### What Makes Them Work?

Universal joints: what angular speed is the output for a constant angular speed input?

What problems are being solved by different kinds of hinges?

###### Newspaper Cuttings

Subscribe for 7 days, save 33%; subscribe for 6 days, save 25%; subscribe for 2 days, save 20%

By buying own-brands and shaving £10 a month off grocery bills, an average household can spend £5 extra on clothes and hobbies and a further £8 on eating out, entertainment & holidays.

Drug O reduced average time to alleviation by 0.68 days; Drug Z reduced average time to alleviation by 0.71 days  
75mg of O daily was 61% effective compared to placebo; 150mg of O daily was 73% effective compared to a placebo

The average Briton has 3 close friends

60.6% of Twitter’s 10m users are over 35 years old; .4% of Facebook’s 200m users are over 35 years old  
Average Facebook user has 120 friends on the site  
More than 3.5 billion minutes are spent on Facebook each day; More than 20 million users update their statuses at least once each day

Nectar has 18.5M cards, used 24 times/sec and in 10yrs £10Bn

###### Modelling

What effect does stride length have on time taken to run a (short) race?

Design an umbrella to keep all of you dry. What if the wind is blowing the rain in your face.

### Consecutive Numbers

###### Two Numbers

I have written down two whole numbers that differ by 2. I multiply them together and add 1.

What properties must my answers have?

###### Four Numbers

I have written down four consecutive whole numbers.

I multiply them together and add 1.

What properties must the answer have?

###### Consecutive Addition

If you add any three consecutive numbers, their sum is always divisible by 3.

If you add any four consecutive numbers, their sum is always divisible by …? Generalise.

In how many different ways can a given whole number be the sum of consecutive whole numbers?

Generalise the facts that 1 + 2 = 3; 4 + 5 + 6 = 7 + 8; 9 + 10 + 11 + 12 = 13 + 14 + 15.

### Marbellous Relationships

###### Marbles 1

If Anne gives one of her marbles to John, they will have the same number of marbles.

What can you say about the number of marbles they started with?

Buses

If one person got off the first bus and got onto the second, there would be the same number of people in both buses.

Ages

If A was one year older and B was one year younger, then A and B would be the same age.

###### Marbles 2

If Anne gives one of her marbles to John, they will have the same number of marbles;

if John now gives two of his marbles to Kathy, they will have the same number.

What can you say about the relation between Anne’s and Kathy’s marbles to start with?

Bus or Age versions?

###### Marbles 3

If Anne gives John one of her marbles, she will then have one more than twice as many marbles as John then has.

If John started with 12 marbles, how many did Anne start with?

Buses or age versions?

###### Marbles 4

If Anne gives John one of her marbles, she will then have one more than twice as many marbles as John then has. However, if instead, John gives Anne one of his marbles, he will have one more than a third as many marbles as Anne then has. How many marbles have they each currently? Buses or Age versions? Generalise!

### Other Mathematical Contexts

###### Triangle Construction

Given a number of equal length sticks, how many different triangles can be made using all of the sticks, where the length of each edge of the triangle is a whole number of sticks?

###### Square Construction

On a grid of dots of given size, how many squares can be constructed? What areas are possible?

What about an equilateral triangle?

###### Arithmetic in Action

There are 5 cups in a row labelled *A, B, C, D, E.* In order to eliminate one, you count along the row then back again (*A, B, C, D, E, D, C, B, A, B, …*). You eliminate the 500th cup. Then you repeat with the remaining cups. Which cup is the one not eliminated?

2*P* people are in a circle. You count around the circle and eliminate every *p*th person until only *P* people are left. Where do you place your *p* – 1 friends so that they and you are not eliminated? [Josephus problem]

###### Repetition

You are told that both of the following two patterns show at least two repetitions of a repeating pattern. Will the 100th square in either be shaded or clear?

 

Where will the 100th shaded square appear in each sequence? Generalise.

###### Understanding Division

234234 is divisible by 13 and 7 and 11;

What is the remainder on dividing 23423426 by 13? By 7? By 11?

Make up your own!