# Ideals, Realities & Actualities

## epistemic stances, affective realities and ontic possibilities
## a contribution to envisioning The Neue Klein Project


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# Flection & Re-Flection

Taking the perspective of the lived experience of teachers and students

# Outline

- Multiple Visions of Neue Klein as presenting:
  - contemporary mathematics to teachers
  - mathematics as Problem Solving
  - mathematics as Explaining Phenomena
  - ways to Bridge the School-University Divide
  - a Unification of Mathematics and its Didactics
- Epistemological Stances
- Affective Actualities
- Ontic Realities





A few words about some, more about others

Each is its own genre in terms of style of writing, intended audience

# Plea

- What ever choice is made of genre and content
- Make use of, call on, activate and draw upon
  - all aspects of the human psyche:
    cognitive, affective and enactive dimensions
  - social embeddedness
  - attention and intention





Ulrich pointed to a quote from Klein:

# Vision 1: Current Mathematical Scene

- Presenting Contemporary Mathematics to Teachers (and others)
- Issues
  - accessibility: to whom with what background?
  - appreciation? understanding? with how much detail?
  - pedagogical and didactic issues (model exposition?)
  - scope: sources within and outside academic mathematics





Taking form of expository articles

# Vision 2

- Mathematics as a human endeavour to identify and resolve classes of problems through understanding underlying structural relationships
  - mathematics as modelling (Lesh)
  - mathematics as problem solving





Mathematics presented as attempts to resolve (and to understand the resolutions of) classes of problems.

Multitude of resources, like multitude of texts ... its what you do with the that matters

# Vision 2 Example

- Imagine a collection of identical regular tetrahedra
- Imagine glueing them together face to face to form a chain
- Can you ever complete a ring, glueing the last to the first?
- even if you can 'cut through' others in the ring?





This is meant as an example of how mathematicians often solve problems ... by converting them into other problems involving different entities

# Resolution: transformations

- Present the first tetrahedron in Affine 4-space
- Glueing is then a transformation representable by nice 4 x 4 row stochastic matrices (involutions)
- Ring exists if non trivial product is identity (group of matrices is not a free-product)
- Replace specific coefficients (2/3) by a letter 'x'
- Entries are now polynomials, and in sequences of products degree grows, polynomials have certain properties
- No possibility of fractional roots
- No non-trivial product of matrices can be identity
- Ring not possible





What is needed by the reader to see this as an example from which a taste of method is appreciated?

# Vision 3: Exploiting Powers & Themes

- Exploiting the immense powers to imagine and to express in language [geometry & algebra]
  - Klein: attempting to unify these modes of experience
- to specialise & to generalise
- to conjecture & to convince
- to restrict and to extend
- to characterise and to classify





Content could be organised by and chosen to illustrate how themes and powers arise and are used in mathematics

# Vision 4: Phenomenal Mathematics

- Mathematics as a means of making sense of phenomena (material, virtual)
  - Conjecture: every mathematical topic and concept can be introduced and experienced through phenomena (material, virtual) which raises a surprise, something to explain.
  - Many times we have been shown virtual phenomena here
  - Issue is developing disciplined way of working ON phenomena [link with vision 2]





By phenomena I mean animations, some static pictures, some material-world phenomena.

Could be presented through surprising phenomena and how they can be accounted for mathematically

Mathematicians as sense-makers, making sense using mathematics and making sense of mathematical relationships

# Vision 5: Bridging School-University Divide

- Showing
  - what can be done in schools
  - what can be done in university
- to promote mathematical thinking AND learning how to learn mathematics





Stressing how topics in school are related to topics in University and vice versa

# Vision 6: Unifying Maths & its Didactics

- Every concept has its informal and formal aspects
- Every action (technique, procedure, practice) has its effects (strengths and weaknesses)
- Problems, Tasks, Phenomena … initiate use of powers BUT
  - need to sustain that energy
  - need to learn from the experience arising from the activity by drawing back from action and re-constructing or re-flecting; what worked, what might work again (utility)





Focusing on the extra that is needed beyond knowing definitions, lemmas theorems and techniques so as to promote learning

# Suggestion

- Each vision entails a different genre of book or other object
- Why not assemble examples of books in different genres and look for what is missing?
  - new genre perhaps?
  - some lacunae?
  - fresh voice?
  - novel unification?





In retrospect, perhaps these different visions can be amalgamated through the use of meta-commenting, but I am not sure about it. To me they presume distinctive ways of thinking about mathematics, and distinctive ways of setting about writing.

# Issues

- Illustrative & Unifying Examples
  - Illustrative for whom?
  - Example for whom?
  - What do you expect readers to do with examples?
  - Pedagogic-didactic issues of exemplification





Whatever format is used, it is vital to work on what the reader is expected to do with an example … so that it becomes exemplary of something more general.

# Epistemic Stances

- It just is
- It looks right
- I've been told ...
- Empirically it seems ...
- It must be (is deducible from ...)
- Social responsibility and opportunity to amplify the empirical and the structurally deductive





Opportunity for mathematics to play a role in the development of the whole person. needs explicit work, as too many students do not seem to grasp the deductive dimensions of mathematics even in high school

# Attention

- Holding Wholes (gazing)
- Discerning Details
- Recognising Relationships (in situation)
- Perceiving Properties
- Reasoning on the Basis of Agreed Properties





Current conjecture: these can go a long way to explain phenomena such as
"students not grasping ideas"
"students not thinking before acting (also affective dimension)"
"miscommunication in classroom"

# Affective Actualities (motivation)

- Disposition (to see mathematically)
- Propensities (to dive in abruptly, to give up prematurely, ...)
- What is role of context?
  - Authenticity?
  - Familiarity?
  - Use situation details to locate structural relationships?
  - To answer "why are we doing this?"
- Encouraging learners to make significant (mathematical) choices





One aspect of the psyche fundamentally different to Klein's audience. Need to engage learners, engage their powers, treat them as choice-making individuals and social groups.

Context is NOT the answer ... it means too many different things to too many people and is not universally effective. What IS effective is stimulating learners to use their own powers; e.g. constructing mathematical objects for themselves

# Ontic Realities

- Students constructing mathematical objects for themselves
  - as part of their exploration, investigation, inquiries
  - as part of their consolidation of technique & understanding
  - as part of their self-assessment
  - as part of their official assessment





Mathematics is a Constructive Enterprise … let it be displayed this way!

# Summary

- Multiple Visions, each requiring a different genre
- To produce a book that has to be highly selective I recommend accumulating a wide scope and then selecting (ie start with the Wiki)
- Make use of the full structure of the human psyche: cognition, affect, behaviour, social embeddedness, attention and intention
- Pay particular attention to what makes an example an example for a reader
- Ways of Working ON phenomena, examples etc.