

# A Study of the Movement of Attention

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# Outline

- Some experiences
- Some reflections
- Why is mathematical reasoning (proof) so hard to teach?
- Why is there so much miscommunication in mathematics classrooms?
- Why do errors appear, and why is learning not always steady and progressive?

# Attention

- When working on the tasks which follow, try to trap the nature of your attention and its movements

# Copperplate Calculation

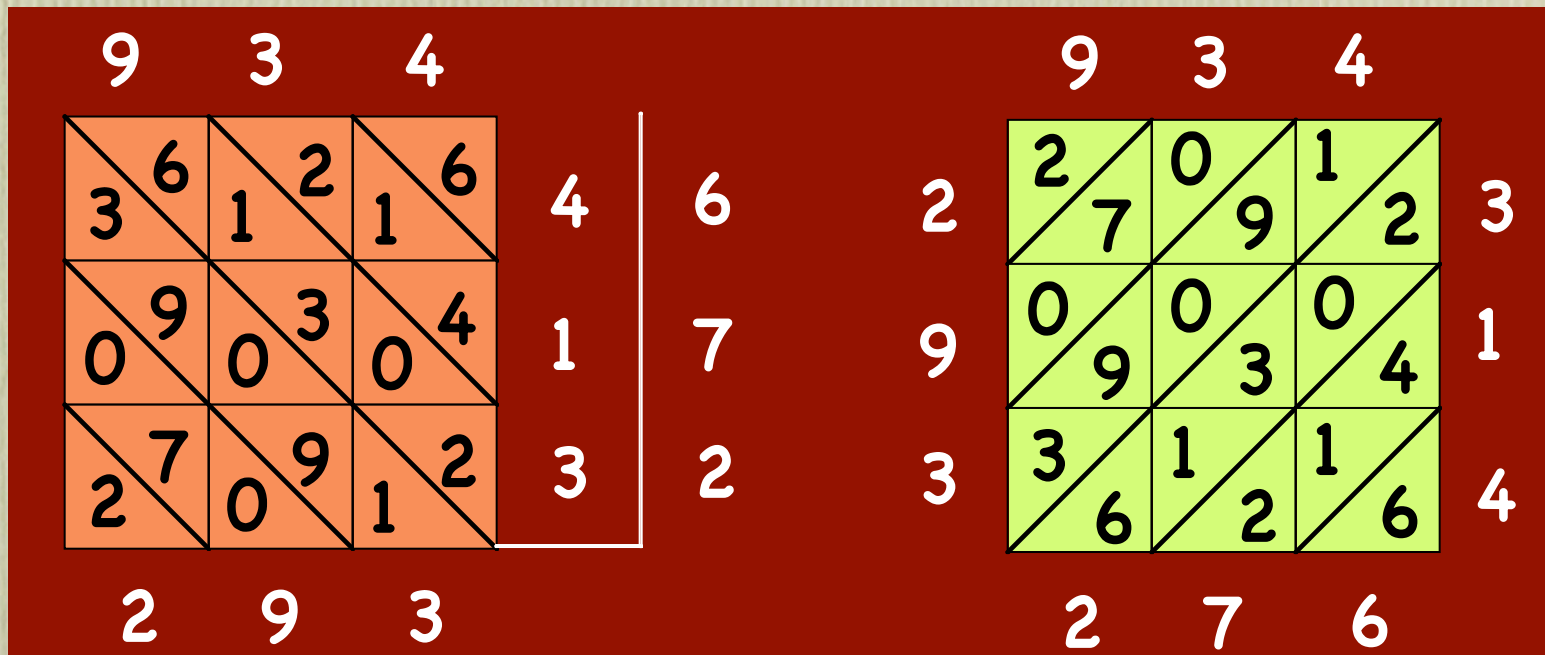
$$\begin{array}{r} 79645 \\ \underline{64789} \\ 30 \\ 2420 \\ 361635 \\ 54242840 \\ 4236423245 \\ 28634836 \\ 497254 \\ 5681 \\ 63 \\ \hline 5160119905 \end{array}$$

# Hindu-Arabic Arithmetic

$$\begin{array}{r} 48 \\ \cancel{5}6\cancel{2}5 \\ \phantom{\cancel{5}}\cancel{8}39 \end{array} \quad \begin{array}{r} 79 \\ \cancel{4}8\cancel{2}5 \\ \phantom{\cancel{4}}8\cancel{3}9 \end{array} \quad \begin{array}{r} 86 \\ \cancel{4}7\cancel{9}5 \\ \phantom{\cancel{4}}83\cancel{9} \end{array} \quad \begin{array}{r} 4786 \\ 839 \end{array}$$

- The calculation comes from an Arabic manuscript Hindu Reckoning written by Kushyar ibn-Lebban about 1000 C.E. (quoted in NCTM 1969 p133)

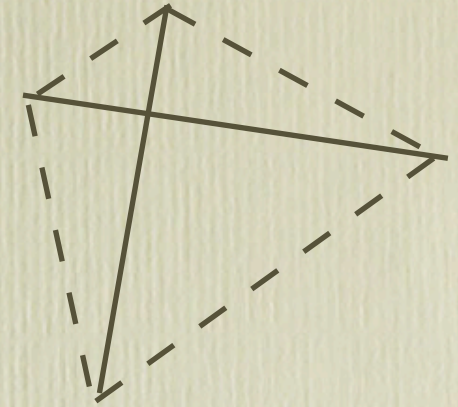
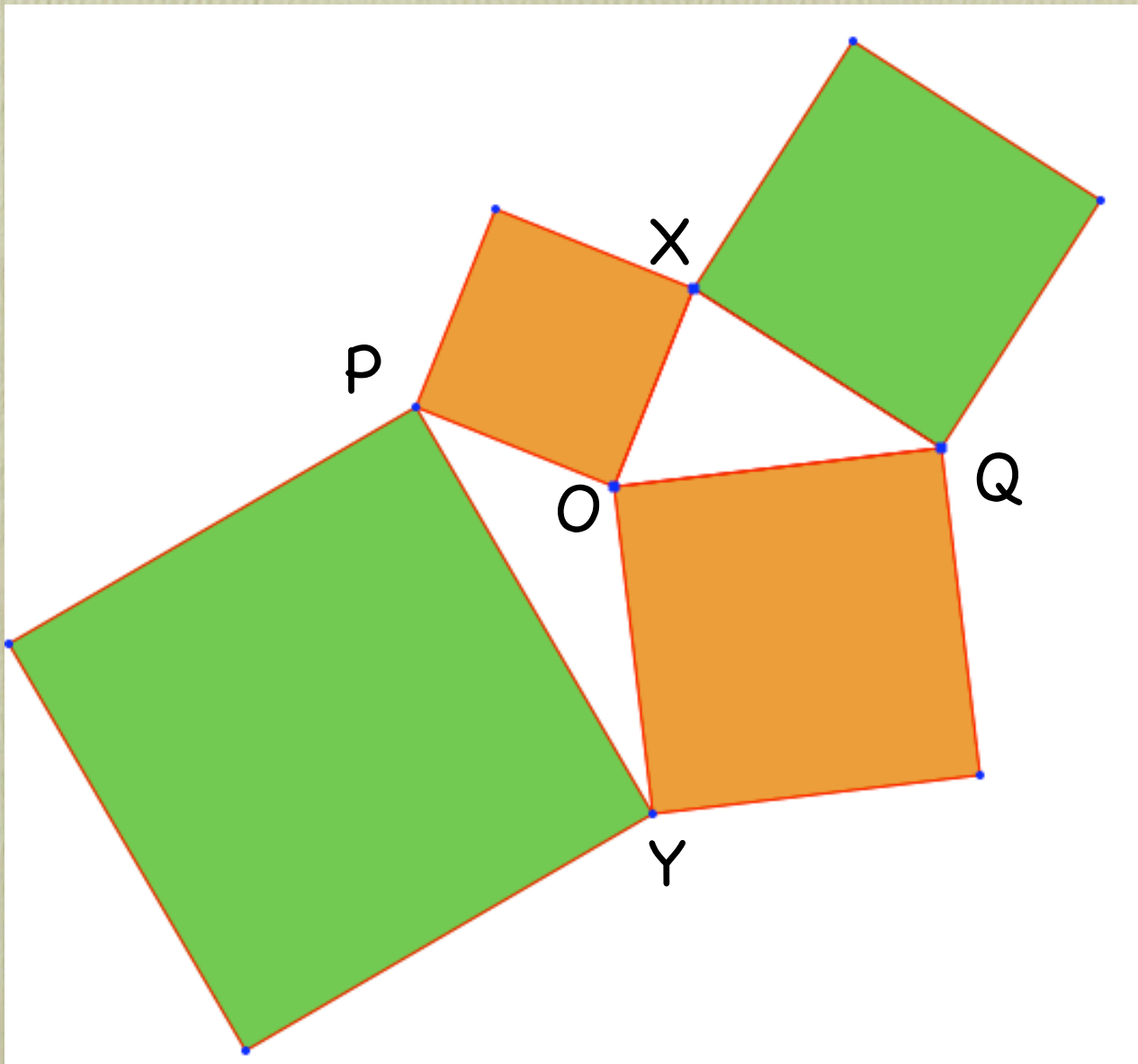
# Same & Different



Treviso and Pacioli Multiplications

Historical Topics for the Mathematics Classroom, NCTM p134.

# Geometrical Configuration



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The two triangles have the same area

The sum of the two green squares is twice the sum of the orange squares.

rotate the orange squares  $90^\circ$  about O anticlockwise

X  $\rightarrow$  P and Y  $\rightarrow$  Q So XY perp to PQ

So PXQY has perp diagonals, which means that  $PX^2 + QY^2 = XQ^2 + PY^2$

So twice the orange squares equals the green squares

# Micro-Structure of Attention

- Holding Wholes (Gazing)
- Discerning Details
- Recognising Relationships
- Perceiving Properties
- Reasoning on the Basis of Properties  
(deducing from definitions)



# Conclusions

- Subtle movements of attention take place all the time. This could help account for
  - confusions and miscommunications in the classroom
  - classic errors made by learners
  - difficulties in teaching reasoning
- to learn to reason you have to be thinking in terms of properties and using mathematical structure and logic as your warrant to justify conjectures
- to reason formally you have to use only publicly agreed properties