RESEACH FORUM FOLLOWUP

Participants in the Research Forum were invited to submit proposals for ways in which they could see themselves pursuing some of the issues raised in the presentations and in the background paper, as well as comments and observations about the issues.

SESSION 1

Contexts in which the Role and Nature of Examples is Worthy of Attention

Classroom teaching

Teacher education (mathematical examples; topic-related didactical choices; pedagogical tactics to choose; …)

Curriculum materials construction

Use of Examples

➢ To initiate discussion amongst students (in what way is this an example?; how could it be modified and still be an example?)

➢ Asking "What makes a (particular) example pedagogically useful?" and "What do I need to do alert students to what it exemplifies?" are useful discussion starters.

➢ Different representations afford greater or lesser access to examplehood according to the knowledge, experience and background of the students.

➢ Preservice teachers working in groups to develop several different approaches to a mathematics topic commonly experienced as 'difficult'could :

- work on developing a sequence of examples for these lessons/approaches.
- critique one anothers' examples
- work on ways of drawing attention to those things they are (and want their students to) attend to.

➢ Getting students to construct examples or to describe what is exemplary can be used to 'determine' or reveal subject matter knowledge and pedagogical knowledge;

➢ Getting students to construct examples or to describe what is exemplary can be used for probing into the understanding of mathematical concepts.

➢ In my research, examples are critical because they must embody not only particular pedagogical techniques but they must promote the mathematical development. Additionally the work of Krutetskii plays an important role in the use of flexibility, reversibility and generalization. Further the projected instructional sequence may impact the type of task with respect to the placement of the task. Finally Vygotsky's influence in providing a zone of proximal development and scaffolding (with some influence of dissonance) also applies. Examples are critical.

➢ Examples are very important for the process of learning. In order to lead to a formulization, the examples have to be very carefully constructed. We must strive to ensure that our students extract from our examples what we intend for them to extract. In our work we have asked students to give examples – a task which is difficult for them.

➢ Working with Grade 8 students with Cabri 3D, giving interesting objects (examples) to inspire students' own creations. Major issue to find examples that move students forward. Random 'play' with software and instructions are both of limited use – examples seem best.

➢ I teach pre-service teachers (equivalent to BA(QTS) but secondary). I teach sessions on the use of examples in the mathematics teaching method course. It's quite interesting to hear their thoughts. It will be more interesting after this research forum.

➢ I would challenge the pre-service teacher students to ask themselves

- What is this an example of?
- What is the purpose of this example (why do you use it)?

➢ Contexts for working with examples:

- Data collection tool to study students' understanding
- Assessment tool working with students
- Instructional tool with pre-service teachers

➢ I use examples to highlight known areas of mathematics in which students have misconceptions. ii. Like the fraction example, I often ask students to design introductory lessons on topics I know are difficult for students to grasp. These lessons invariably include examples. iii. Sometimes we compare and contrast examples with respect to a range of factors e.g. suitability for grade level, gender/ethnic sensitivities etc.

➢ In the classroom setting help students to appreciate this strategy

- Offer examples and ask them to notice what is the same and what is different
- Invite them to reflect on each others' examples – what is the power of each example?

➢ In the context of ITE and student teachers, students should be aware of the literature on example and concepts/misconceptions in the particular topic in order to be able to choose examples of the latter.

➢ Everything in math is an example

- a*b = b*a, example
- a+b = b+a, example
- 2+3 = 3+2, example
- 2 apples + 3 apples = 3 apples + 2 apples
- 2 apples together with 3 apples = ……
- So the question is a big one!

➢ Short tests that require giving examples (of objects that) have certain (combinations of) properties.

➢ In my own context – I teach mathematics and can not only devise and use my own materials but can also devise my own syllabus. Thus opportunities for reflecting on examples I use abound. I have come to believe that it is valuable in choosing an example to work on to offer ones which (a) are non-trivial at that point for those students and (b) we work on together for some time.

➢ Pre-service teachers:

- Examples of lesson plans
- Examples of actual lessons

➢ For pre- and in-service (secondary and primary) need to be explicit about purpose of examples. Possible to consider what would be a good and a bad example and why.

➢ I hope I can offer examples which exemplify/point to practices that provide opportunities for all to participate mathematically.

➢ With the use of examples we can build a link between the complexity of the concepts – used on the subjects.

➢ Awaken awareness

- Have teachers discuss good and bad about particular examples
- Have teachers prepare and discuss examples, ask provocative questions
- If possible, show excerpts from teaching and analyse what the examples entail Opportunities for studying the use of examples

- Study students' lesson plans
- Record what comes from examples in my own teaching
- Many – not time

➢ In the formation of student teachers: examples are a good way for establishing their understanding, examples are as well an important means of communicating between teachers and student teachers.

Opportunities to Research in More Detail

➢ Construction of curriculum materials

➢ Ways of interacting with learners in order that they appreciate the examples they have; how are these choices made?

➢ Ways in which teachers can and do spontaneously construct and exploit examples; how are these choices made?

➢ Pre-service maths methods classes – mostly workshop based with groups challenged by examples to deconstruct and reconstruct mathematics in pedagogically useful ways;

➢ Studying examples used by students in response to probes; analyzing tasks/examples from various perspectives – what do students see in/as example, what do teachers see?

➢ Awakening student awarenesses through tasks asking for the construction of examples:

➢ As a Junior High school teacher I can study:

- The types of examples provided by one textbook
- Ho do I choose examples to show my students' what are my criteria?
- How do students use examples to express their thinking?
- Do I / should I encourage students to use/construct examples?

➢ I am interested in how students generate examples. In my research I have advanced calculus students investigating proof tasks. They spontaneously generate examples in their own explanations and justification from definition.

➢ I work in a curriculum research and development centre where there are several opportunities to research further:

- What examples do the researchers/designers use – tied to curriculum theory, research in classes, discipline etc.
- What examples do school-age students use in explaining their thinking about the solution to a problem (we have a laboratory school, grades K-12 that is part of our unit)
- What examples do the teachers use when they need to go beyond the planned curriculum – and under what circumstances do they create new examples?

➢ Opportunities for studying the use of examples:

- Observing student teaching and/or reading lesson plans …encountering examples of examples they use and discussing how they functioned
- Collecting and discussing examples at conferences
- Systematic presentation/discussion in the context of introducing a topic (in PGCE lectures)

➢ In my research I'd really like to look at the examples the students provide as illustrations/exemplifications of generalizations/abstractions. What do they think is a good example and why?

➢ Opportunities for studying the use of examples:

- Any context – assessment .. so examples in summative assessment (tests) need to work without teacher input (purpose of examples in this context – show/illustrate the problem and illustrate how to communicate reasoning)
- Also interest in computer: when pupils work at own pace and use examples without teacher live input
- How to design examples to stand alone without the teacher input?
- Hints/learning activities in software
- Therefore need to be aware of common predicable misconceptions and select examples appropriately.

In my role as head of department and subject mentor opportunities to study use of examples:

- Observing student teachers and discussing with them while planning what examples they use and why. Often use ones from textbooks or ones they feel comfortable with without any real thought as to why
- Lesson observations of both student teachers and other members of the department – feedback discussion after lessons
- Scrutinising new scheme of work (realistic maths in context) with other teachers teaching it. We look closely at the examples, situations and models the scheme offers us to use and the thought the writers put into those.

➢ Observation of trainee teachers with a focus on examples used followed by discussion with trainees and mentors

➢ As a researcher studying beginning in-service secondary teachers' growth in choice and/or construction and use of examples.

➢ As a teacher educator, using examples in teachers' learning and facilitating pre-service teachers' selection and construction of examples/tasks.

➢ Opportunities to study use of examples:

- Pre-service secondary teachers in clinical situations for methods classes.
- Student teachers/first year teachers in middle school and secondary classes.
- Graduate level in-service teachers (from variety of backgrounds)

➢ As a teacher educator I could study the relationship between the "examples" (in general terms) of mathematical and pedagogical situations I offer to my student teachers and the examples they are using in their teaching practice or later in their actual teaching.

➢ PGCE students developing and trialing examples on school experience. Comparative studies of text book exemplification again involving PGCE students. Exemplification of exemplification (modeling exemplification) – its effect on student practices. Categories of exemplification by characteristics – investigating types of dialogue stimulated by types of example. Authority, Democracy and Power: the role of examples in the formation of classroom interactions and relationships.

➢ In pre-service (PGCE) study examples used in seminars, that are used to awaken pre-service teachers' awareness of example construction. Monitoring use – lesson plans, observation of teaching, discussion forums (e-learning).

➢ In connection to reading comprehension and learning by reading examples can be studied as parts of larger texts, and how students use these parts of the text for learning 'in general' or for solving a specific related problem.

➢ I am a teacher so I could look at my own practice and that of my colleagues. I could also make designs where my students have to produce examples. I've done it a few times and view it as a taxing but rewarding activity for the students.

➢ In my context there are a number of possible ways to study the use of examples – primary ITE: student teachers' choices of examples in the lessons that they teach; my own use of

examples to illustrate approaches to T and L of maths; are whole courses for students really constructed of different examples and groups of examples?

- Context 1 – Value and potential of 'impossible' scenarios – make explicit the anticipated learning
- Context 2 – the progression of examples - tending to make learning reduced to small procedural steps but also possible with 'difficult' examples. What questions can you ask that get at the meaning of addition of fractions? Pirie article 2001?

➢ Research into examples: one makes opportunity for one's priorities! (a) study one's <u>own</u> choice of examples (b) textbooks' choices (c) students' constructions/choice of examples – classify nature of these (I do that)

➢ I currently study teachers' uses of representations in practice … this directly leads to examining their use of examples in practice. Do they work out examples of do they have the students create examples? Is there a sequence in how teachers use examples; *when* do they introduce and use them?

➢ In student teaching seminar we might look at videos and ask 'Is this an example of good teaching?'

➢ Find out how mathematicians use examples in their own teaching and in their research. Collaboration with mathematicians on how they choose their examples. Investigate *generic examples*.

➢ In everyday lessons! Little trials.

Other Comments

➢ Your discussion of examples resonates for me the aspect of "basic repertoire" in the framework of seven aspects of teacher subject matter knowledge (Even, 1990). I would be interested to know how these two are similar/different (Ruhama Even)

➢ A teacher listens to students' ideas/misunderstandings and produces an unplanned example to let students correct their misunderstandings. Difficult!

➢ The discussion in the first session [seems to] assume[s] a "pedagogy of illustration". The specific 'examples' discussed originated with the teacher, the audience for which was a class of students (we assume of differing abilities and understandings). The task "suggest three pairs of fractions each of which sums to three quarters" places the onus on the student to generate the examples. This is a different pedagogy!

➢ I'm a bit surprised at the implied pedagogical sequencing here which I interpret to be a movement from formal representation to meaningful possibility via the strategy of pre-selected examples versus moving from experience to mathematical interpretation, allowing the context to contribute to the pool of potential examples.

➢ Have a notion of what we are trying to do. Be open to changing this. Be secure in your relationship with mathematics, teaching and the class. Think about non-mathematical purposes – safety, pleasure etc. Give students a way in (a range of ways in) I am reminded of work I read recently by Sarah J Greenwald on using examples drawn from the Simpsons and Futurama. Also another paper (I can't remember by whom) on using the Klein [?] game as an example for graph theory and use of weighted averages. Context allows much more complex examples to be offered?

➢ I teach calculus at a Department of Engineering. In my context it would be interesting to study the use of examples to understand the concept of derivatives, differentiable functions. And to study why , very often, students take the given examples as the mathematical proof. Which are our responsibility? What do we do wrong presenting examples to students, if they then become to many students, structured mathematical proofs?

➢ In this forum what does 'example' mean? Specifically, is a motivating problem situation an example? (Response: a mathematical example is something mathematical that is taken as

exemplary; an application or context is example if students see it as exemplifying possible occurrences of the topic. Meta Examples might include the nature of examples/expected pedagogy by students.)

- What is entailed in choosing examples? Choice is always filtered through current 'mathematical' awareness. What is being exemplified is current example space.
  - How might (how are) examples be exploited?
  - What are particularly effective ways of working on examples? Start simple? Start complex?
  - What do students see (as variable): constraints? awareness?
  - How do you awaken pre-service/in-service teachers' awareness of the issues, pitfalls and methods of example choice and use? In Dynamic Geometry random examples are often very useful. Examples seen as didactic objects could bring them and their use more to the surface
  - What matters is way of working.
  - Objects are just objects, until someone sees them as exemplary, so exemplariness is a property of the awareness of the person not the object

Specific Comments on the Presentations

- Construction of examples: know your audience in addition to the material being worked with. It is not enough to say the audience is '11 years old" – need more context. What had these students had learned before? Which country are we in?

- Was the teacher giving wrong data in order to evoke discussion? In the fractions example, no reference was made to graphic representation. Teachers try to look for logic example. Logic is dangerous – one starts with visual and/or tactile examples.

- Different levels of sophistication in the excerpts. Congruency: far better for <u>students</u> to construct triangles with the given info, so that two possibilities emerge. Fractions: Need visuals or concrete manipulatives. Confirm/establish equivalent fractions as prior knowledge.

- Extract 2's special case is not illuminating and raises too many questions. Good examples (1) make one clear point (not multiple points) (2) facilitate generalization or establish impossibility.

- Geometric "sense" is needed – diagrams roughly the right measurements. "Impossible" triangles like these can lead to valuable classroom discussion – if the teacher actually realizes they are impossible and has chosen them to illustrate this. I wonder how many people here did not realize they are impossible. We often take things that are written, such as examples in books or examples given us by colleagues at face value and assume there are no problems. Student reactions to different examples used – the nature of the discussion the examples generate.

- i) Both the examples ask for find out more models of triangles satisfying given conditions (ii) in the first example student misses to see that there is no model for the 2nd. He/she fails to see more examples (iii) examples then get students to focus either on theory or on modelisation.

- First example for addition of fractions: Why not a group activity, with pattern blocks liberally provided, to investigate a half plus a sixth as a group task?

- Difficulties in relationship with diagram. Was the teacher's error deliberate? The second example is wrong too.

- In Ex 1 the second case creates self-conflict by the student who applies angel sum of triangle, then visually the angel other than the base angle can't be 100 degrees. In Ex 2 similar experience of conflict would occur. The ?? triangle co-exists nicely.

- The teacher should not give arbitrary elements of the triangle. In the first excerpt the teacher in his attempt to give an example of an isosceles triangle changes the value of the angles arbitrarily keeping the same figure. In the second excerpt the teacher has in his mind a model of the proof that seemed not to be accepted or adopted by the student. Working with pre-service teachers I could study the practices developed and the examples that they chose.

- Give the students a realistic and not trivial problem, like one seventh plus one eighth parts of a cake. Ask them to *estimate* the sum of these two pieces of cake.

- Was the task (1) pre-planned or was it put on the spot? What were the teacher's expectations?

- Both examples lead to cognitive conflict. Difference: example 1 gives <u>incorrect</u> data, example 2 gives correct data. A clear purpose is necessary before preparing examples.

- Some participants were perplexed by the presentation of a teacher constructing a non-existent triangle for students to work on, not perhaps realizing that the point was that examples constructed on the fly may not be very good ones. Note the use made by Guy Broussseau in his presentation, of a 'false' example.

- Story (flavour it according to the age group) Find the finite number of steps to pass the distance of three quarters of a metre. Is this the only way of walking to reach three quarters? Should your steps be the same length? Use the examples generated by the pre-service/in-service teachers to *assess* their content and pedagogic knowledge. The setting of the context they work in is flexible. Peer interviews are very useful in this regard: both interviewer and interviewee will try their best (examples/questions).

SESSION 2

1. If there is something written on the board the students would use this pattern and replace 3 by 5 and q by m and so on. If there is nothing on the board students would just put down majority of relations like 5*7=35 (possibly with remark m=5) or use previous knowledge and write a table which is frequent in our text books, even in class 2.

m	1	2	3	8	6
$5m$	5	10	15	40	30

2. This session has raised my awareness of how we as researchers in our project select examples and it has made me begin to explicate the characteristics of our examples. I plan to develop that fully to include relationships to other learning and instructional cycles. Thanks for a thoughtful session – I'm looking forward to more discussions on this. (Barbara Doherty)

3. What can I do in my position to investigate examples (yesterday's question). The notion of example space seems to be a really useful tool to investigate students' understanding through SGEs.

4. Where is the student/learner voice in this? Could this not be accessed first (after brief intro). Can learn a lot about where to begin from what the students know already. At times things seem top-down. Where is the reciprocity? (What is the didactic contract all about?)

5. If … then (what if it isn't?) vs. when … then. This begs the question "why?" – the task is based on 'abstract mathematical world'. Should examples make connections with a pupil's world?

6. A "seam of examples to mine". A counter-example is more effective if it represents a 'seam' – less easy to 'monster-bar'

7. I am still somewhat confused by what you are calling an example and what I as a curriculum researcher/developer call a task. If there is a distinction between the two I am not sure what that is. The way I would contrast the two is: a task is something I might construct for children to act on/engage in. An example might be that which I construct to represent the concepts students must encounter. I haven't said this very well – still need to mull it over.

8. Who is served in the didactic contract? Who is served by examples?

9. I would like to consider writing a paper that addresses the "prompting' feature of examples in the context of conceptual change (i.e. reflection on activity–effect relationship). This notion – prompt – plays a substantial role in this model. By way of example the prompt "a number that you subtract one and is then divisible by 7" is markedly different mathematically from a cognitive point of view than "number that is one more than a multiple of 7".

10. 1967 Edith Biggs told a story about examples in which a student finally said to the interviewer "how many examples do I have to give before you are satisfied I know what I know" See Mclean and Biggs "Freedom to Learn" 1967, Addison Wesley.

11. It might very well be that students formalized 3q to a rule of multiplication, not that q is an open place (variable) in which you can substitute. In other words this lesson was about arithmetic, not about algebra. The student that could not find a number that left remainder one on division by 2 – is it not possible that she tried to find out what the interviewer wants – her question cannot be that easy!!

12. Can the word 'example' be isolated from the phrase 'an example of something for somebody'? It may be that the production of examples is the control act of the mathematical imagination. I find that a discussion of the human imagination is greatly missing from this discussion of examples.

13. How about this example? Changing exponential to log form and vice versa b to the power $y = x$ implies log to base b of $x = y$. Example log to base 2 of 4 = 2 change to exponential 2 squared = 4 (then learners are asked to try an exercise) Now try: log to base 3 of 9 = 2 change to exponential.