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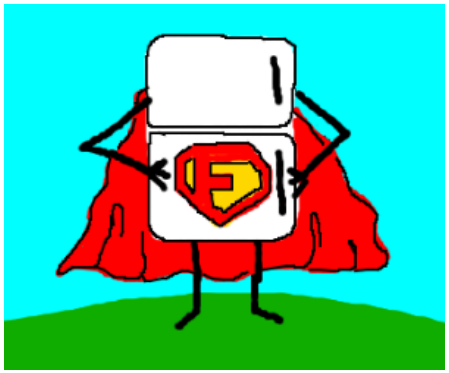
Controlling a large population of smart refrigerators as a leaky storage unit

Simon Tindemans, Vincenzo Trovato, Goran Strbac



http://drawception.com/viewgame/tGKT5dGMF/superfridge-and-his-magnetic-sidekicks/

Workshop on the Mathematics of Demand Side Management and Energy Storage, 2nd June 2015, The Open University

Related publications

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- **Decentralised control of thermostatic loads for flexible demand response.**
  Simon H. Tindemans, Vincenzo Trovato, Goran Strbac
  IEEE Transactions on Control Systems Technology, (2015)
- **The Leaky Storage Model for optimal multi-service allocation of thermostatic loads.**
  Vincenzo Trovato, Simon H. Tindemans, Goran Strbac
  In review
- **Security constrained economic dispatch with flexible thermostatically controlled loads.**
  Vincenzo Trovato, Simon H. Tindemans, Goran Strbac
  IEEE PES ISGT Europe 2014, Istanbul

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MARIE CURIE ACTIONS

The power of fridges

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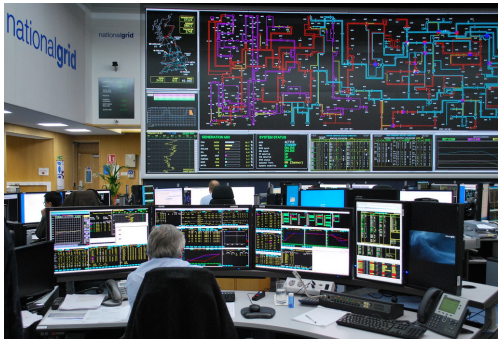
- Decreasing system inertia
- Fewer sources of frequency response
- Flexibility/ramping constraints

Problem



Solution

- Always-on load of significant magnitude
- Flexibility due to intrinsic thermal storage

Requirements and challenges

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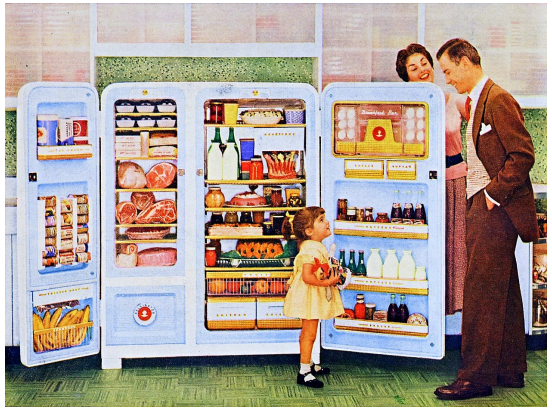
System operator

- Predictability of response
- Stability of control scheme
- Ability to request a range of services depending on prevailing system conditions (frequency response, reserve, ramping, ...)

Commercial operator

- Reliability of contracted response
- Avoiding expensive real-time command and control infrastructure
- Ability to estimate available flexibility

End user



- Don't interfere with cooling ability
- Avoid extra effort and cost

"Lloyds Building, London - 2007" by Diliff - Own work. Licensed under CC BY-SA 3.0 via Wikimedia Commons

Elements of the control scheme

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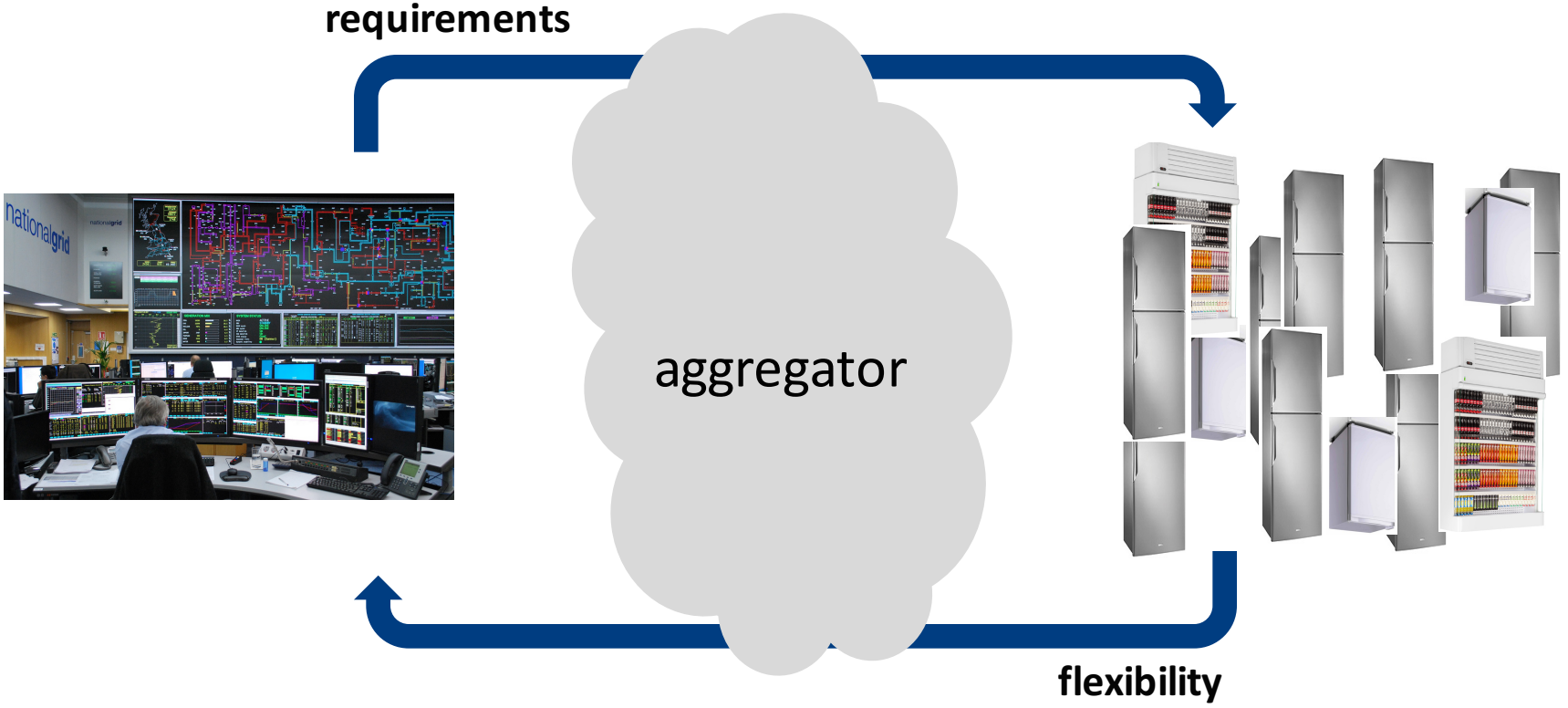
1 **Decentralised control scheme**



2 **Aggregate flexibility model**

The basic fridge model

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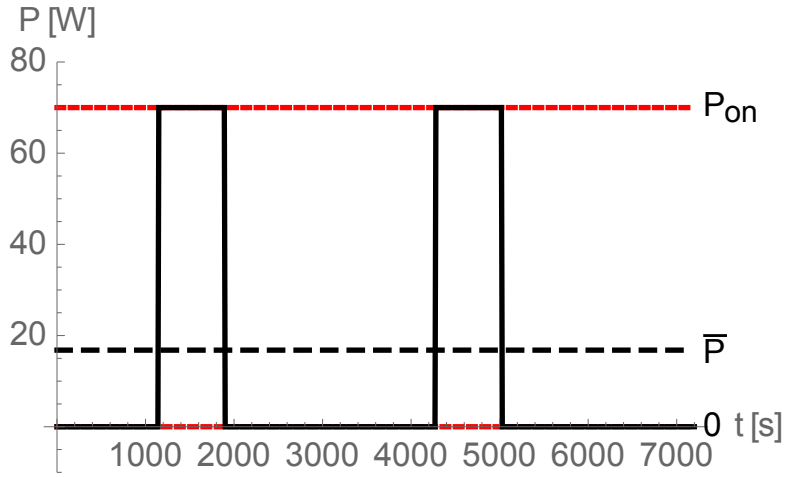
Power consumption:

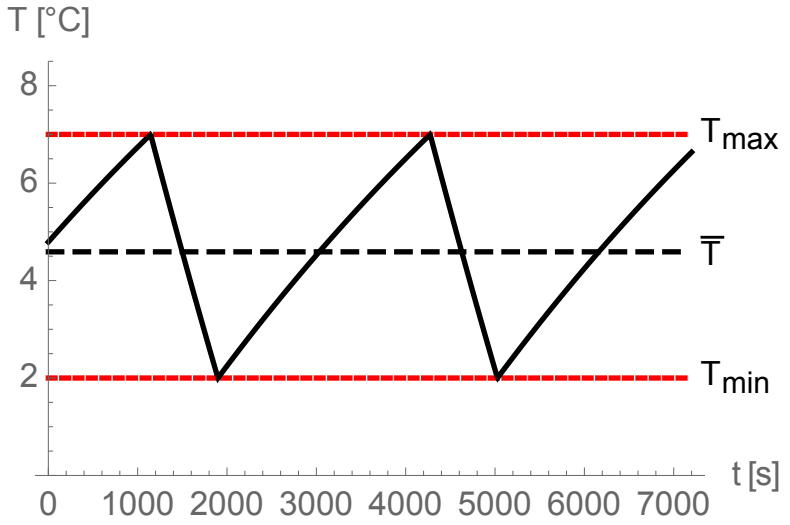
$$P^a(t) = \begin{cases} P_{on} & \text{if device is on} \\ 0 & \text{if device is off} \end{cases}$$



First order linear model for the temperature

$$\frac{dT(t)}{dt} = \begin{cases} -\alpha(T(t) - T_{on}) & \text{if device is on} \\ -\alpha(T(t) - T_{off}) & \text{if device is off} \end{cases}$$

Hard temperature limits at T_{min} and T_{max}



Probabilistic control of power consumption

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It is sufficient to control the *expectation value** of the power consumption of each appliance:

$$E[P^a(t)] = \overline{P_0^a} \times \Pi(t)$$

control signal
$\Pi(t) = 1$ for steady state
modulates power consumption of all appliances

actual power consumption of appliance *a* (a random process)

steady state power consumption of appliance *a*

Result

$$P^{total} = P_0^{total} \times \Pi(t) + \mathcal{O}(N^{-1/2})$$

*: expectation with respect to random initial conditions and control actions

Inverse control via distribution

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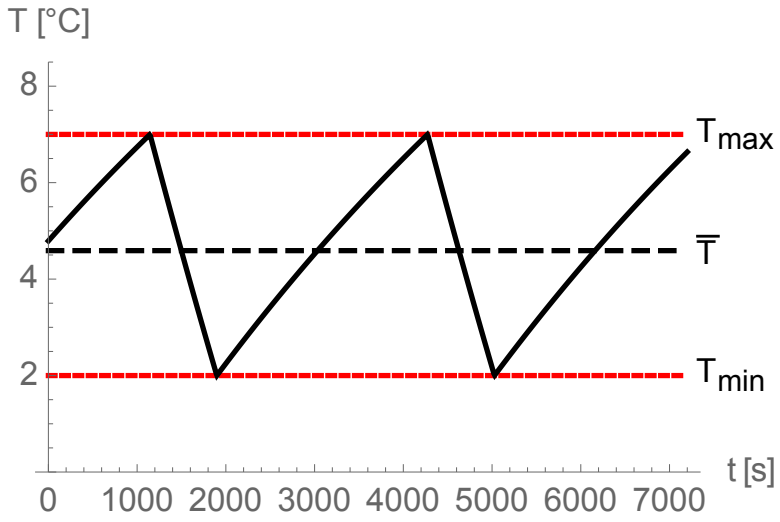

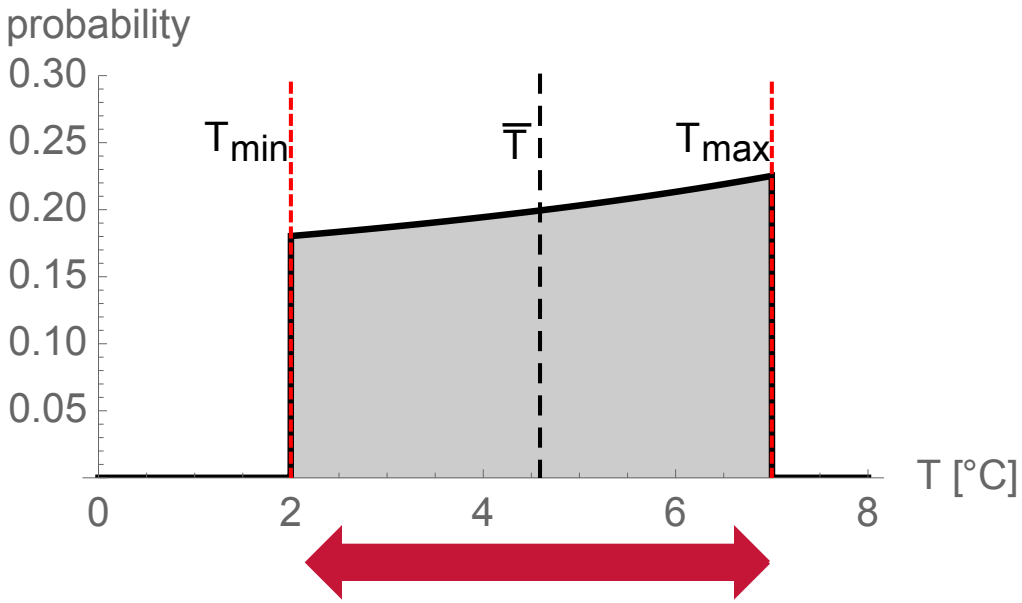

Compress or stretch temperature distribution so that

- Temperatures never exceed T_{min} and T_{max}.
- Compression/expansion is 'sufficiently slow' to be realisable.
- Average temperature changes as: $\frac{d\bar{T}(t)}{dt} = -\alpha\left[\bar{T}(t) - T_{off} + \left(T_{off} - \bar{T}_0\right)\Pi(t)\right]$

desired power level

Implement local controller to realise this distribution

- Deterministic + stochastic switching

The control process

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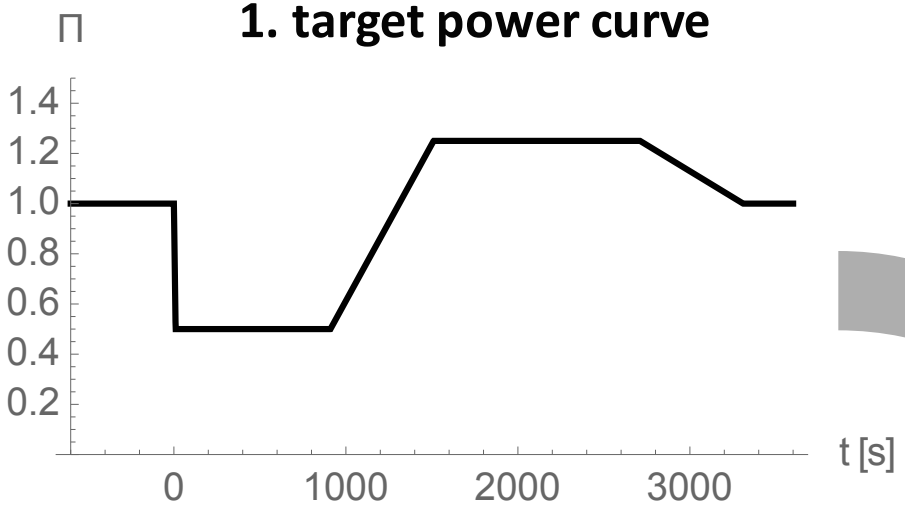
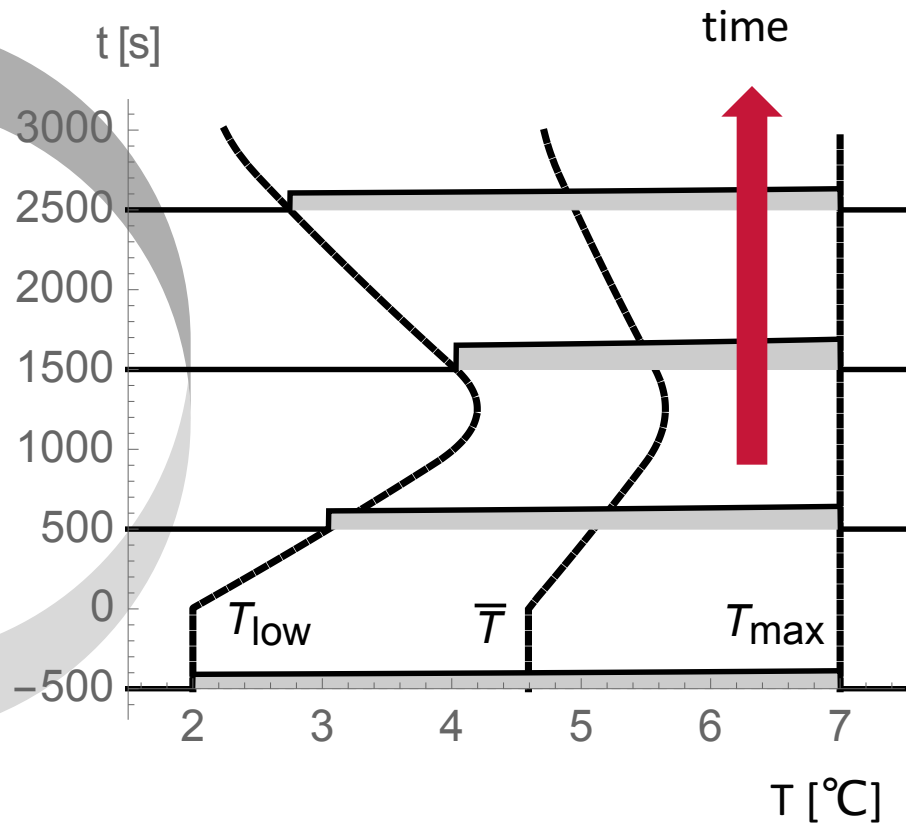
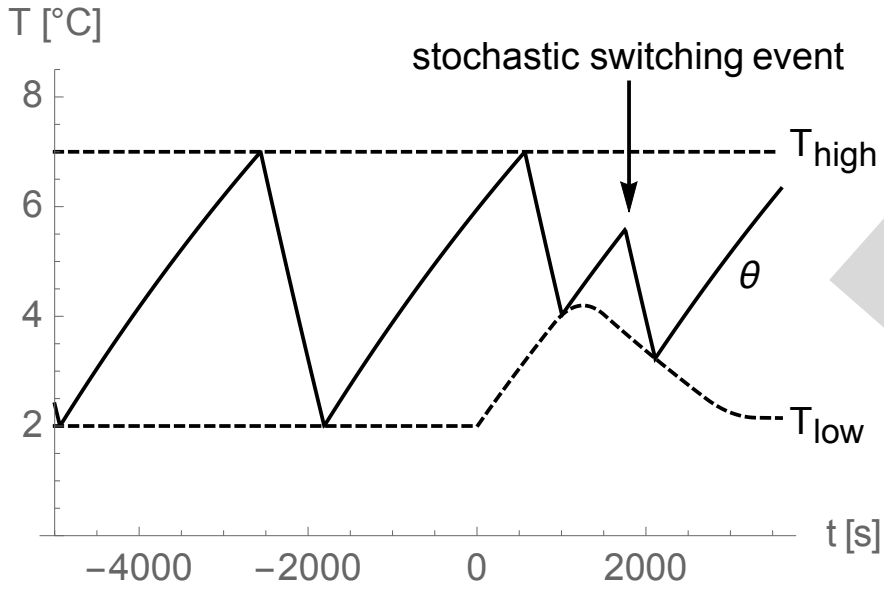
1. target power curve



2. evolution of temperature distribution



3. individual appliance switching



Case study

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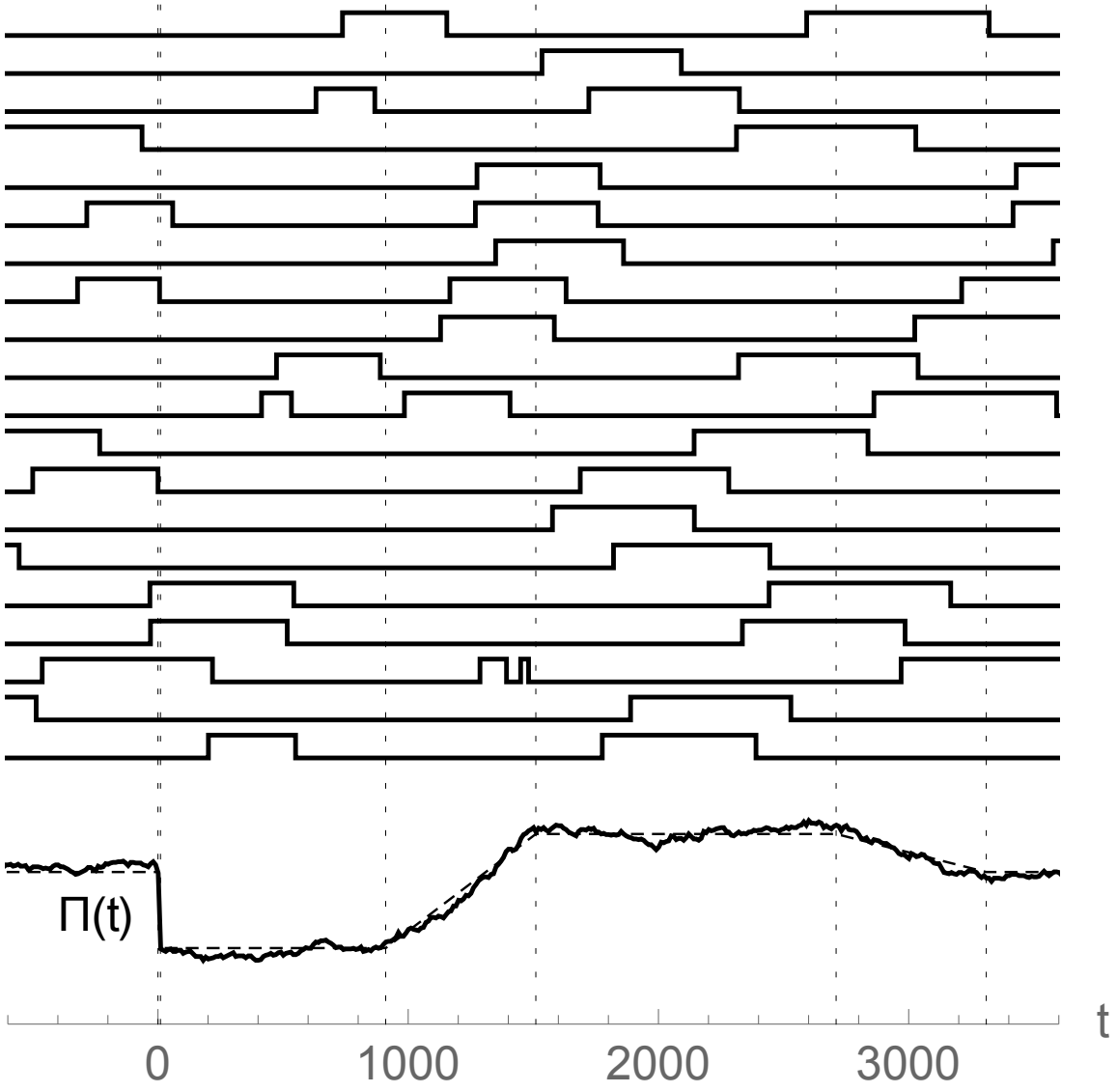

N=20 identical refrigerators compute independent control actions as a function of $\Pi(t)$.

N=1000 identical refrigerators

Step 2: summarising flexibility

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1

Decentralised control scheme

- Direct control of aggregate power consumption
- Respects temperature limits

$\Pi(t)$

control signals

aggregator

flexibility

2

Aggregate flexibility model

What are the limits to controllability?

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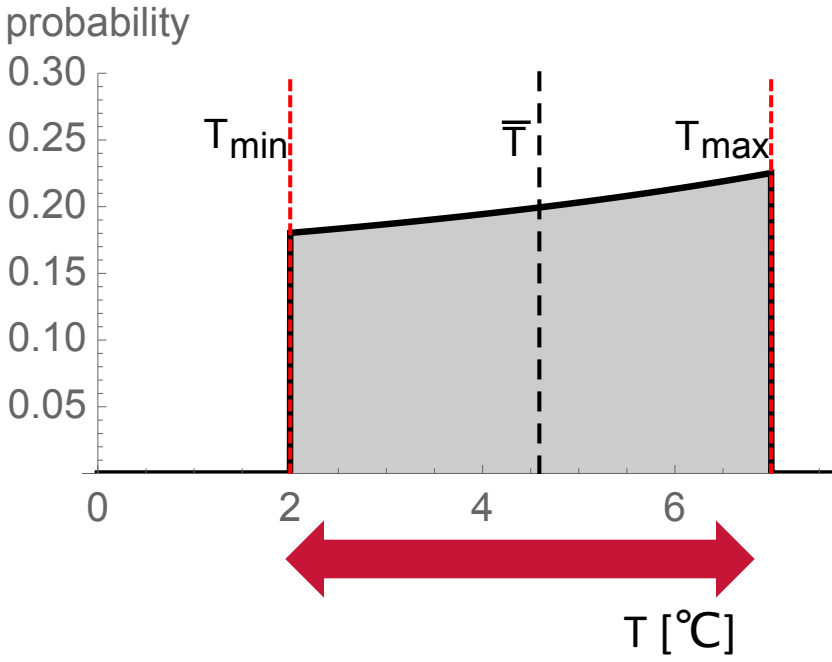
"How long can a response be sustained?"

Effective **energy constraints** are determined by temperature limits. Example: $\bar{T}(t) \in [2,7]°C$

$$T_{min} \leq \bar{T}(t) \leq T_{max}$$



"What is the accessible range of power consumption levels?"

Determined by constant* **power constraints.** Example: $\Pi(t) \in [0.44, 2.44]$

$$\Pi_{min} \leq \Pi(t) \leq \Pi_{max}$$

$$*: \Pi_{min} = \left[\frac{(T_{off} - T_{min})(\bar{T}_o - T_{max})}{(T_{min} - T_{max})(T_{off} - \bar{T}_0)}\right]$$

$$\Pi_{max} = [T_{min}(T_{max} - T_{off}) + \bar{T}_o(T_{on} - T_{max}) + T_{max}(T_{off} - T_{on})]/[(T_{max} - T_{min})(T_{off} - \bar{T}_0)]$$

The leaky storage model

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Create an explicit link to storage systems by the coordinate transformations:

$$S(t) = \left(\frac{\sum P_0}{\alpha}\right)\frac{T_{off} - \bar{T}(t)}{T_{off} - \bar{T}(0)}$$

$$P(t) = \Pi(t)\sum P_o$$

Then the aggregate system dynamics are:

$$\frac{dS(t)}{dt} = P(t) - \alpha S(t)$$

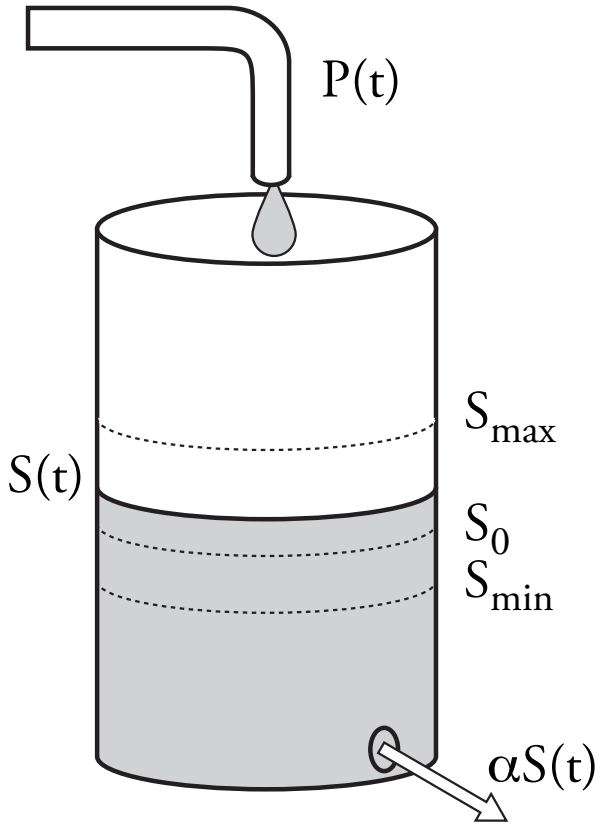
with constraints:

$$P_{min} \le P(t) \le P_{max}$$
$$S_{min} \le S(t) \le S_{max}$$

preserve the food! →

$$\int_0^T S(t)dt = S_0$$

The leaky storage model



End-to-end availability of flexibility

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$\Pi(t)$

control signals
or control models

1

Decentralised control scheme

- Direct control over aggregate power consumption
- Respects temperature limits

aggregator

device parameters

2

Leaky Storage Model for flexibility

- Sufficient and accurate linear model
- Applies to **heterogeneous populations**

Optimal service allocation
Case study: 59,524 refrigerators (1MW)

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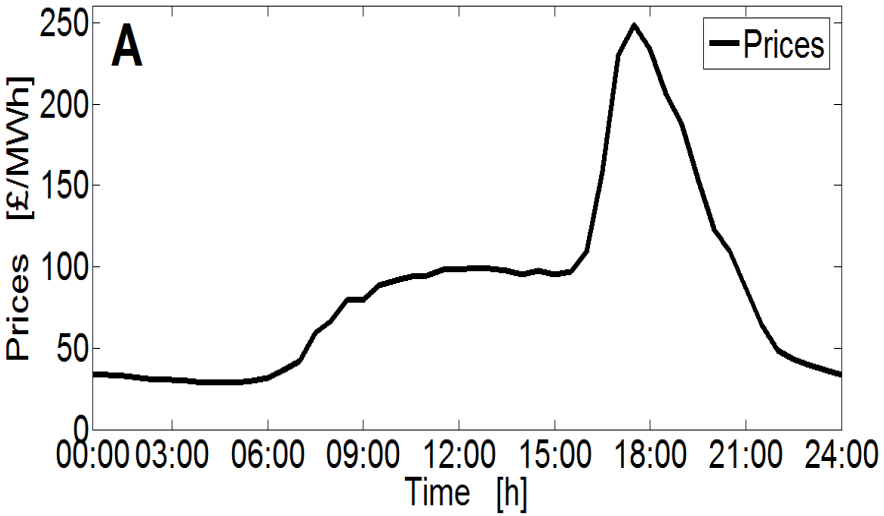
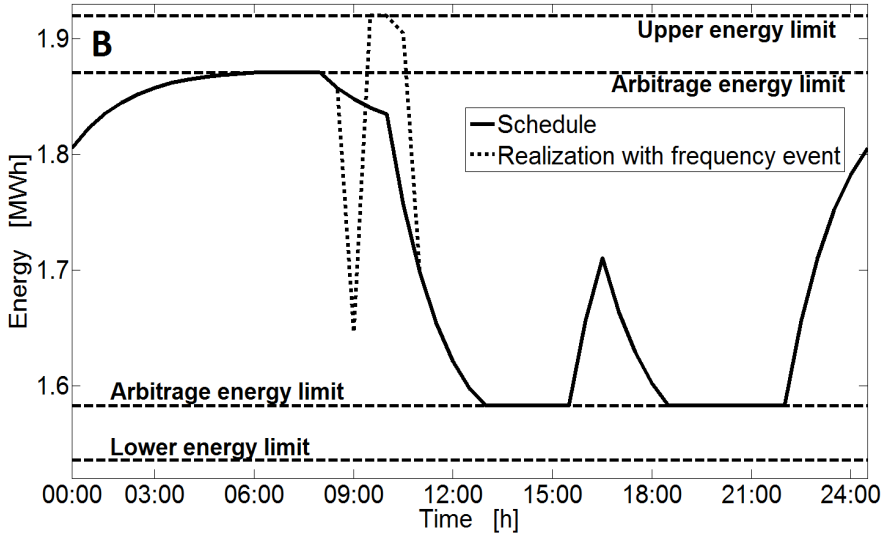
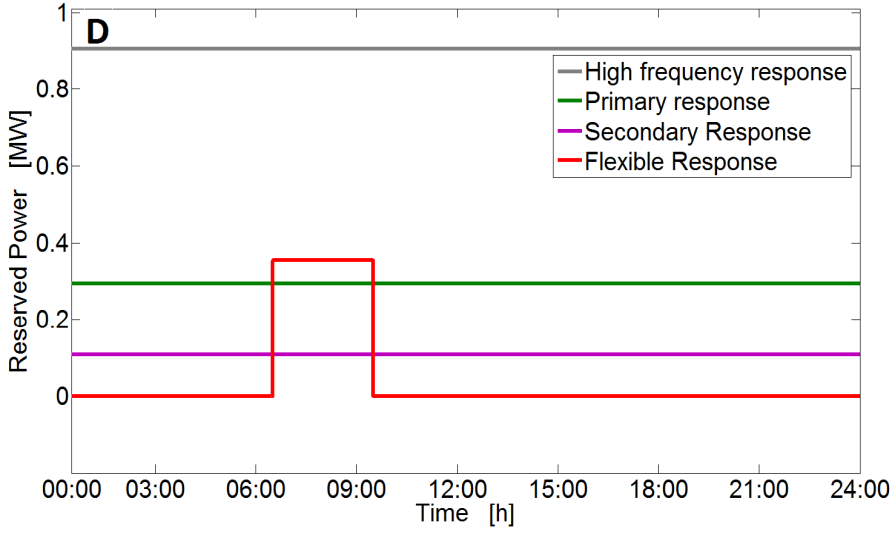
Price levels



Energy schedule

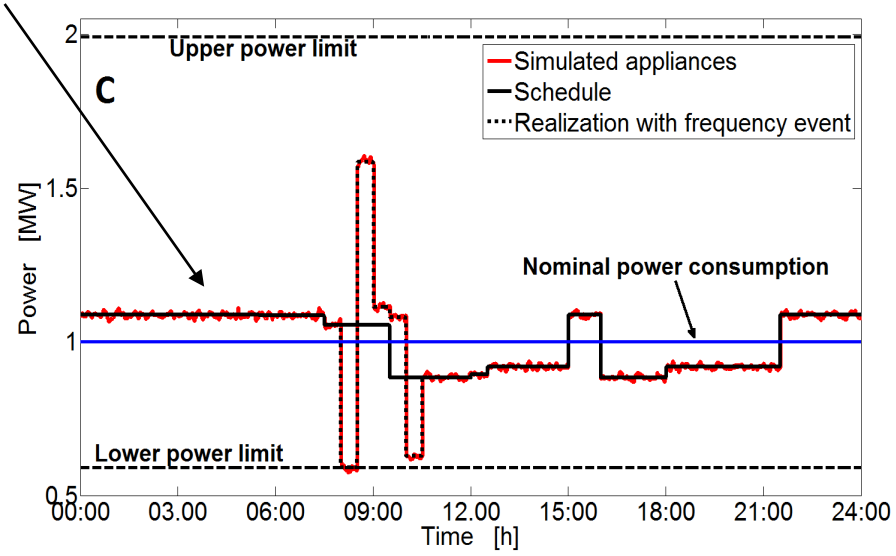


Ancillary service allocation



simulations

Power schedule



Different devices, different services

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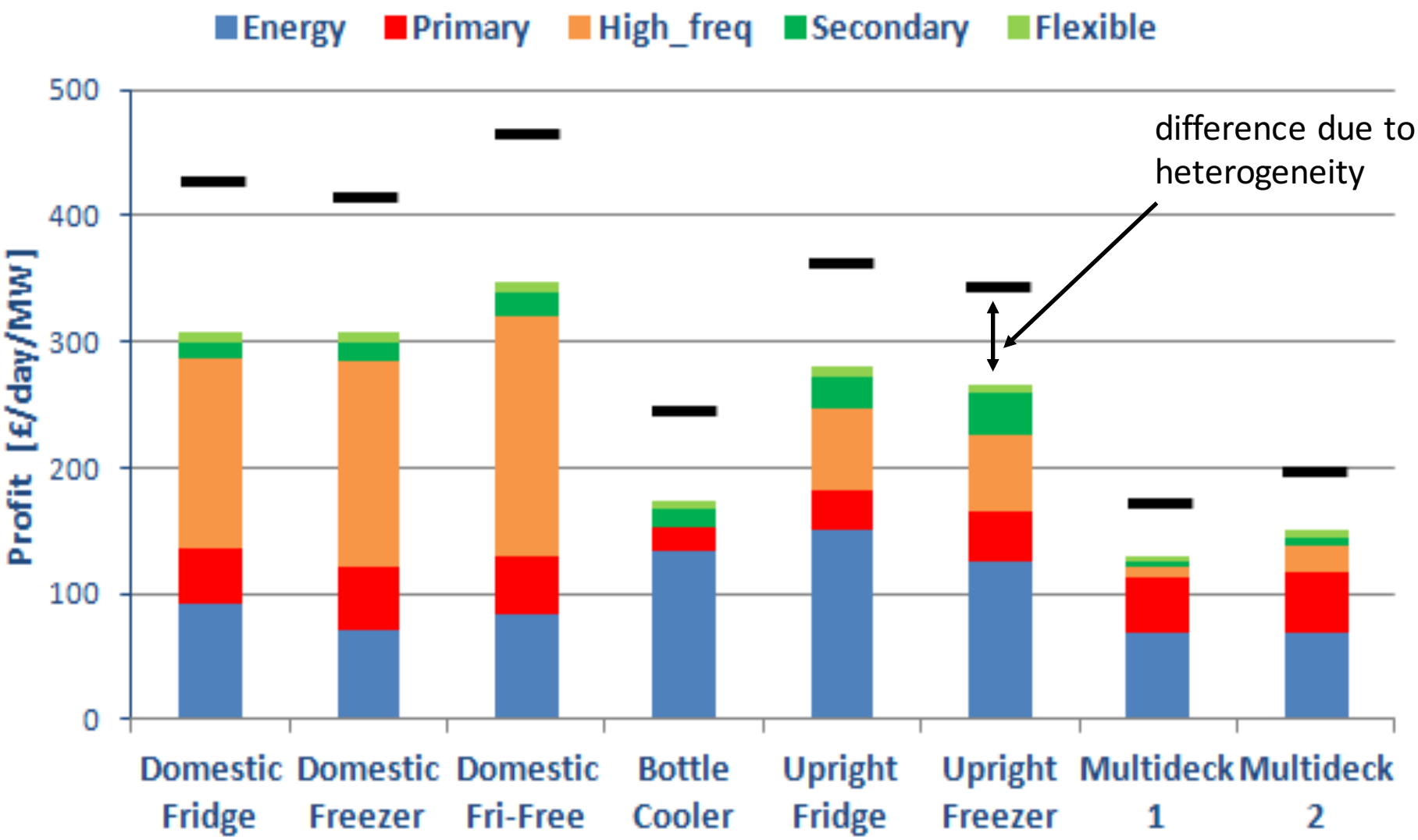

Summary

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Decentralised control scheme

- Respects device-level temperature limits at all times
- Does not require real-time communication
- Precise control over aggregate power consumption
- Distribution-centric: device-level control actions are derived from desired population-level results.

The leaky storage model for aggregate flexibility

- Intuitive summary of flexibility
- Linear model for easy embedding in optimisation
- *Sufficient* model for guaranteed achievability
- Can be used for heterogeneous appliance clusters

Example to show benefits accrued by multi-service allocation for different appliances.