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Mean-field game formulations for distributed storage management in dynamic electricity markets

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Overview and Motivations

- Domestic micro-storage devices are considered: they charge/discharge energy from the network during a 24h interval, trying to maximize profit

- ADVANTAGES:

  1) Profit for the users

  2) Benefits for the system (i.e. reduction in demand peaks)

- MAIN PROBLEM: management of the devices

  i.e. : if they all charge when price is low → shifting of peak demand

- PROPOSED APPROACH:

  - model the problem as a differential game with infinite players (Mean Field Game)
  - solve the resulting coupled PDEs and find a fixed point

1. Modelling

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Modelling: storage device

The single storage element is modelled as:

$\dot{E}(t) = u(t)$ $E:$ *Charge of the device*

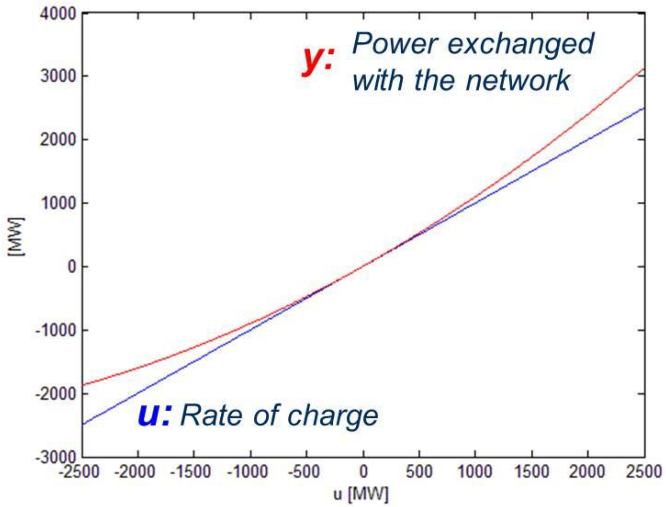
$u:$ *Rate of charge*

- The quantity of energy that can be stored in each device is limited: $0 \le E \le E_{MAX}$

- Maximum rate of charge (u_{MAX}) and discharge (u_{MIN}) are set: $u_{MIN} \le u \le u_{MAX}$

To model efficiency, quadratic losses are introduced:

$$y(t) = u(t) + \gamma u^2(t)$$



We assume that the number of devices is extremely high and can be approximated as infinite → The charge status of the population is described by the distribution function: $m(t, E)$

The optimal control will be calculated in its feedback form → $u^*(t, E)$

Modelling: demand and price

- We consider an original profile for (inelastic) demand **$D_0(t)$**, known without uncertainties.

- Power exchange between the devices and the network is modelled as a variation of demand:

$$D(t) = D_0(t) + D_{ST}(t) = D_0(t) + \int_0^{E_{MAX}} m(t,E)\left[u(t,E) + \gamma u^2(t,E)\right] dE$$

D_0 : *Original demand profile*

D_{ST} : *Variation introduced by storage*

- The price of electricity is assumed to be a monotonic increasing function of demand:

$$p(D(t))$$

Mean field game approach

- In a certain time interval T_{END}, each device exchanges energy with the network, aiming at maximizing its profit:

OBJ. FUNCTION: $$J(E_0, u(\cdot)) = \int_0^{T_{END}} p(D(t))y(t)dt + \underbrace{\Psi(E(T_{END}))}$$

(to minimize)

MEAN FIELD GAME

Additional term to impose desired final charge

Coupled Partial Derivative Equations:

1. **Transport equation**: describes the evolution in time of the distribution ***m(t,E)*** of the charge level

$$\partial_t m(t,E) = -\partial_E \left[m(t,E) u^*(t,E) \right] \qquad m(0,E) = m_0(E)$$

2. **HJB equation**: returns the optimal cost-to-go function ***V(t,E)*** and the optimal control ***u*(t,E)*** of the devices for all time instants and charge levels

$$-\partial_t V(t,E) = \inf_u \left[p\left(D_o(t) + \int_0^{E_{MAX}} m(t,E)\left(u^*(t,E) + \gamma u^{*2}(t,E)\right) dE \right)(u + \gamma u^2) + \partial_E V(t,E) u \right]$$

$$V(T_{END}, E) = \Psi(E)$$

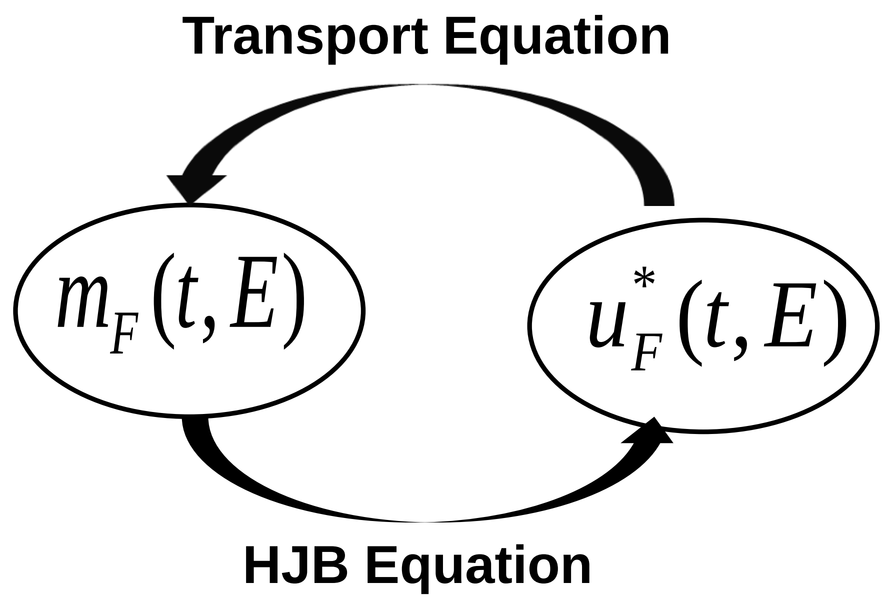
2. Existence of solution

Fixed point

The two PDEs are interdependent:

1. The transport equation has to be integrated forward and depends on the optimal control *u*(t,E)* returned by the HJB
2. The HJB equation is integrated backward and depends on the prices of energy, that are affected by the energy distribution *m(t,E)*

OBJECTIVE: find a couple $(m_F(t,E),u^*_F(t,E))$ which represents a fixed point solution for the PDEs



Existence of solution

We are interested in conditions of existence and uniqueness for the fixed point.

Two different approaches:

1. **Constraints on the state E are temporarily removed:**

- The problem is considerably simplified as a differential game.
- Under mild assumptions, it is proved that **a fixed point exists and is unique.**

2. The original MFG problem is considered:

- Different approaches have been adopted:
    - *Application of existing theorems for MFG.*
    - *Prove the differentiability of the cost-to-go function V in the HJB equation.*
    - *Apply Pontriagyn Minimum Principle with state constraints.*
- For the time being no conclusive results have been achieved

3. Numerical simulations

Iterative algorithm

1) The HJB equation is integrated starting from V(T,E)=Ψ(E) and assuming a known distribution $\overline{m}(t,E) = m_0(E) \quad \forall t \in [0,T]$

At each time step i:

1.1) Initial estimate $\overline{u}(t,E)$ is calculated assuming price $p_{IN}(t) = p(D_0(t))$

1.2) The price is updated: $p_{FIN}(t) = p\left(D_0(t) + \int_0^{E_{MAX}} \overline{m}(t,E)[\overline{u}(t,E) + \gamma \overline{u}^2(t,E)]dE \right)$

1.3) Steps 1.1 and 1.2 are repeated until convergence

2) Once the values of *V* and *u** have been calculated, a new estimate for *m* is obtained integrating the transport equation

3) Steps 1 and 2 are repeated until convergence of *V* and *m*

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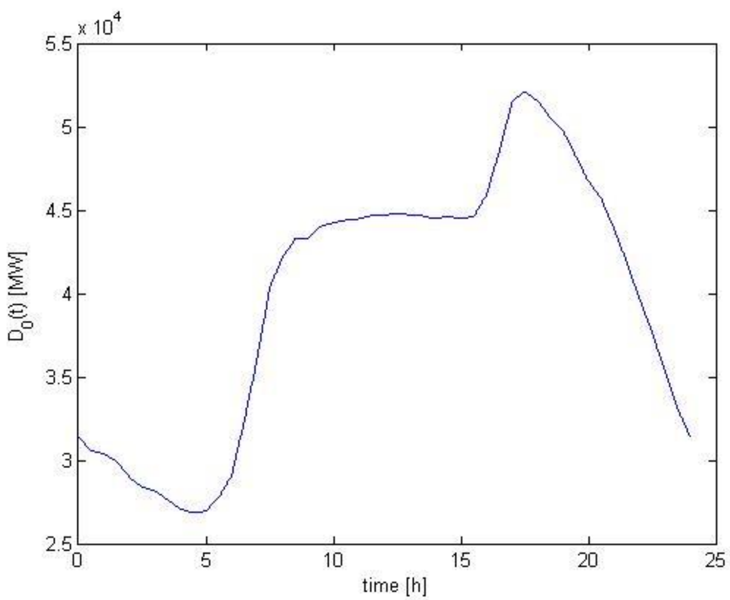
Parameters

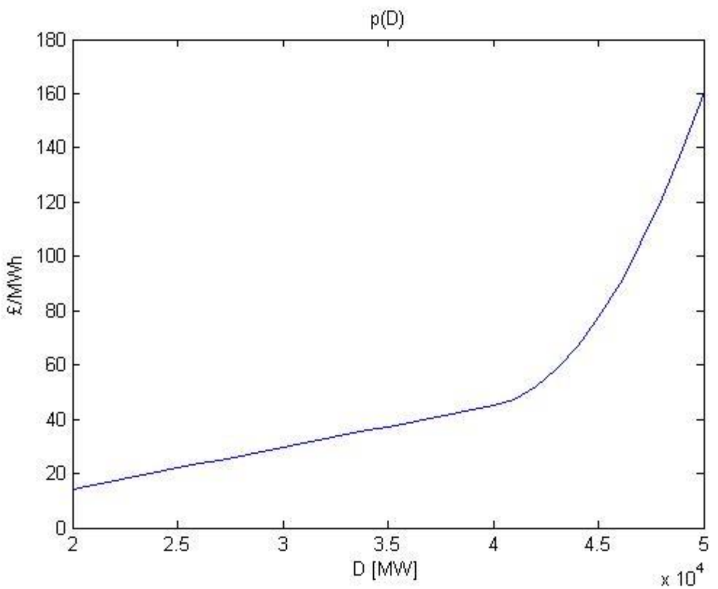
- Typical UK demand profile
- Total storage capacity: $E_{MAX} = 25GWh$
- Each device can fully charge/discharge in 10 hours: $u_{MAX} = \frac{E_{MAX}}{10h} = 2.5GW$

$$\Psi(E) = c\left(E - \frac{E_{MAX}}{2}\right)^2 \qquad \Delta t : 0.1h \qquad \Delta E : \frac{E_{MAX}}{1000} \qquad \gamma : 10^{-4} \qquad T_{END} : 24h$$

DEMAND PROFILE



PRICE FUNCTION



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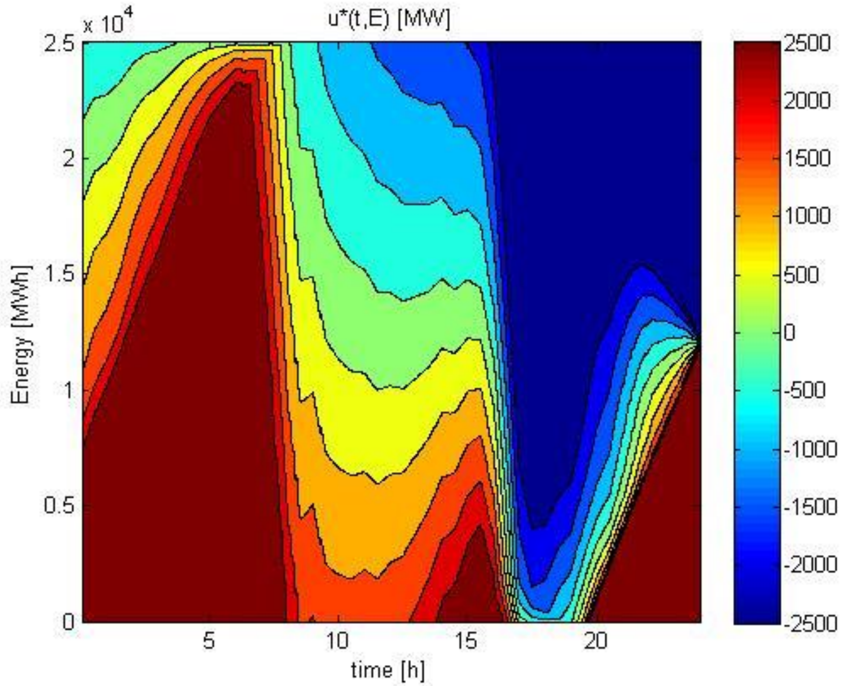
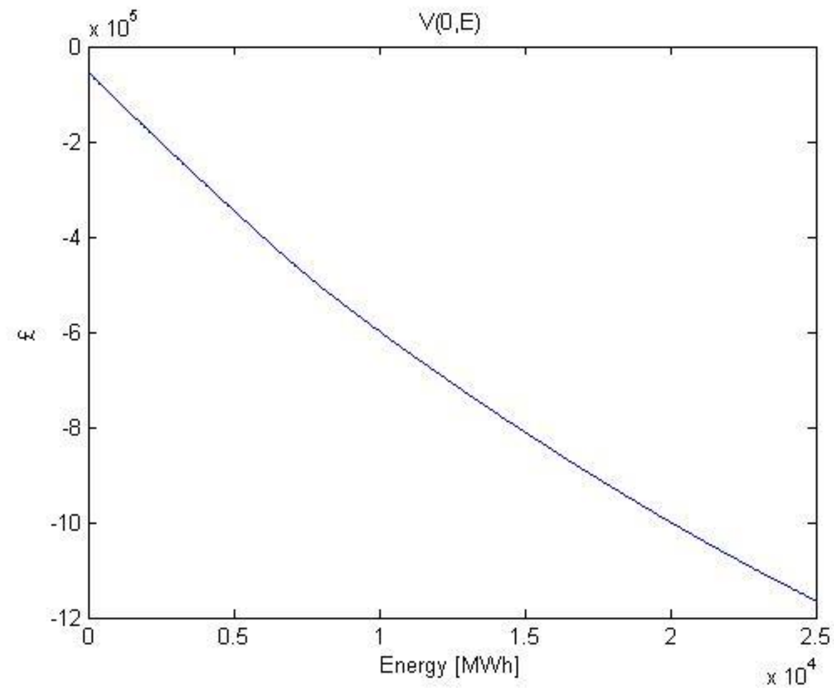
Simulation results (1)

Optimal control *u*(t,E)*



Optimal cost-to-go function V(0,E)



NOTE: the optimal control and the cost function are calculated in the MFG framework and they refer to the ***whole population***. The values for the single devices are obtained dividing by the total number of players.

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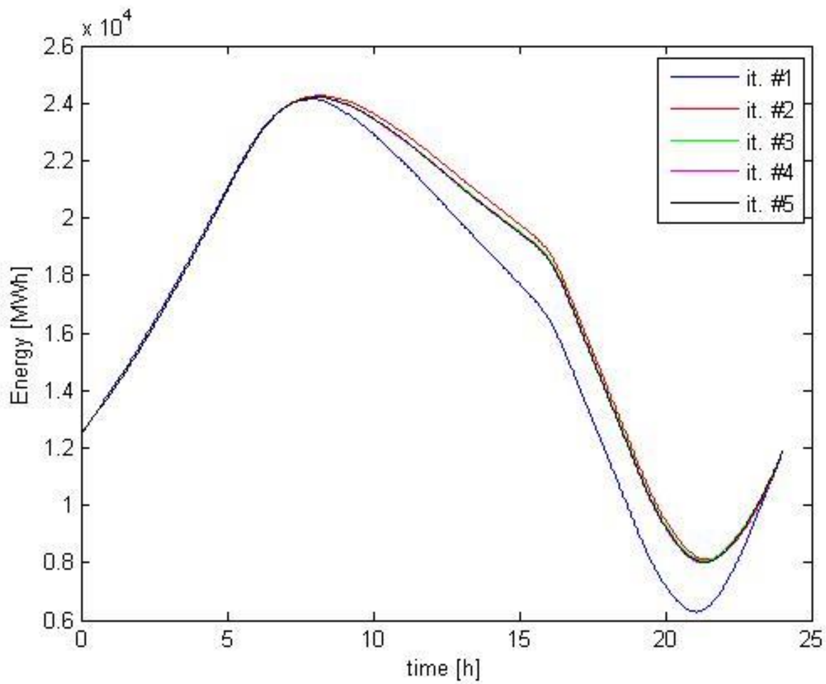
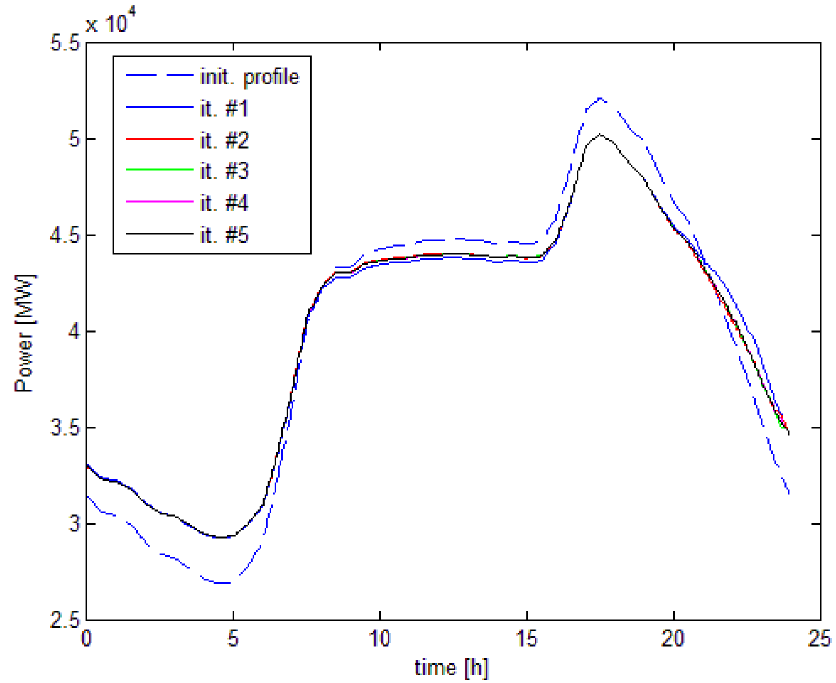
Simulation results (2)

TOTAL STORED ENERGY:



Total energy stored in the devices at each iteration of the forward/backward integration: a fixed point is reached

DEMAND PROFILES:



The storage devices are able to considerably reduce the amplitude of peaks and valleys in the original power demand profile

4. Cyclical constraints

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Periodic constraints (1)

SO FAR: we have operated on the final cost-to-go $\Psi(E)$

$$\Psi(E) = c\left(E - E_{DES}\right)^2$$

All devices will have the same final energy level E_{DES}

$$m(T_{END}, E) \cong \delta(E_{DES} - E)$$

We ideally want: $m(0, E) = m_0(E) \qquad m(T_{END}, E) = m(0, E)$

Same charge distribution at the beginning and at the end of the considered time interval

NEW APPROACH:

A cyclical cost function is introduced: $\Psi(E) = c(E - E(0))^2$

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Periodic constraints (2)

We introduce a new state variable I(t) in the HJB eq.

$$I(t) = \int_0^t u(s)ds$$

The single device is now described by:

$E(t):$ *Current energy level (to take constraints into account)*

$I(t):$ *Total variation of energy (we want $I(T_{END})$ to be small)*

The HJB equation now becomes:

$$-\partial_t V(t,E,I) = \inf_u \left[p^*(t)[u+\gamma u^2] + \partial_E V(t,E,I)u + \partial_I V(t,E,I)u \right]$$

$$\Psi(E,I) = c \cdot I^2$$

← *By adding the state I(t), we can **explicitly penalize** differences between initial and final state*

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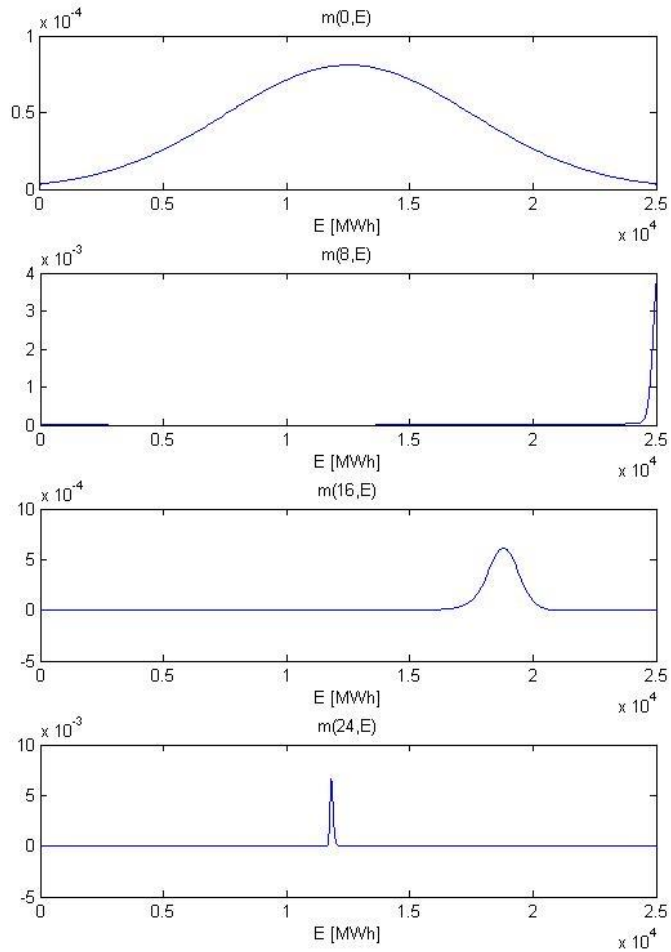
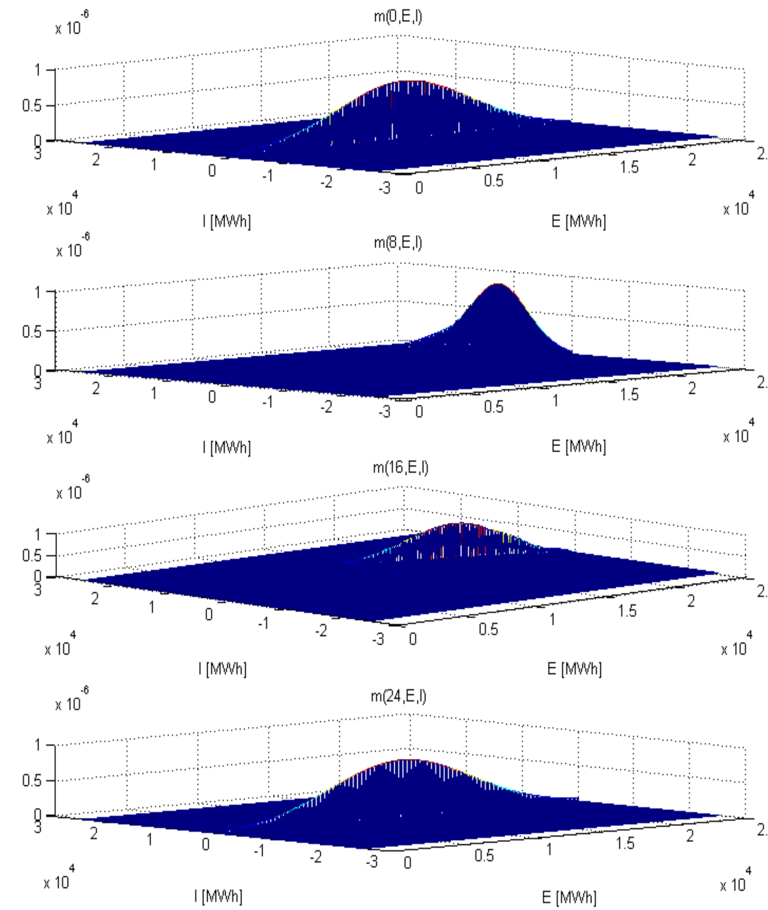
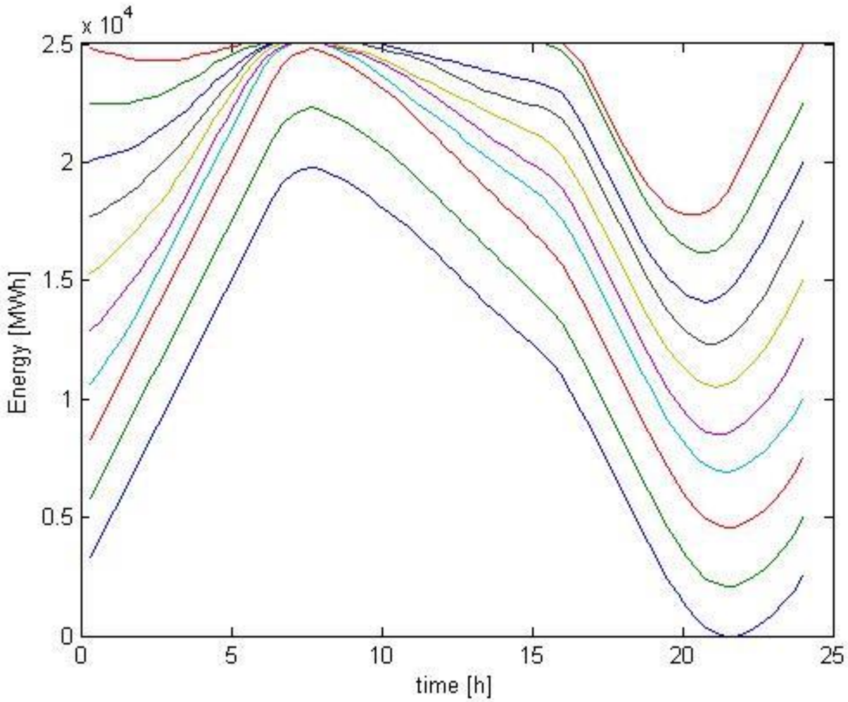
Simulations (1)

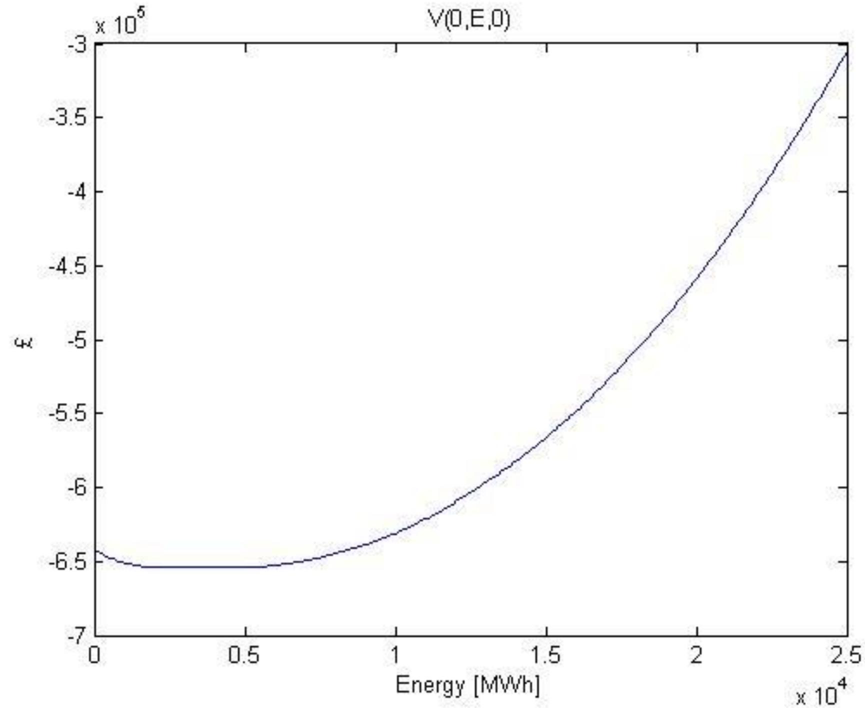




Simulations (2)



Energy level of the storage devices for different initial energies (last iteration)



Values of the optimal cost-to-go function *V(0,E,0)*. Lower values are achieved for devices with low *E(0)*, which are able to charge more energy in the initial phase, when prices are lower

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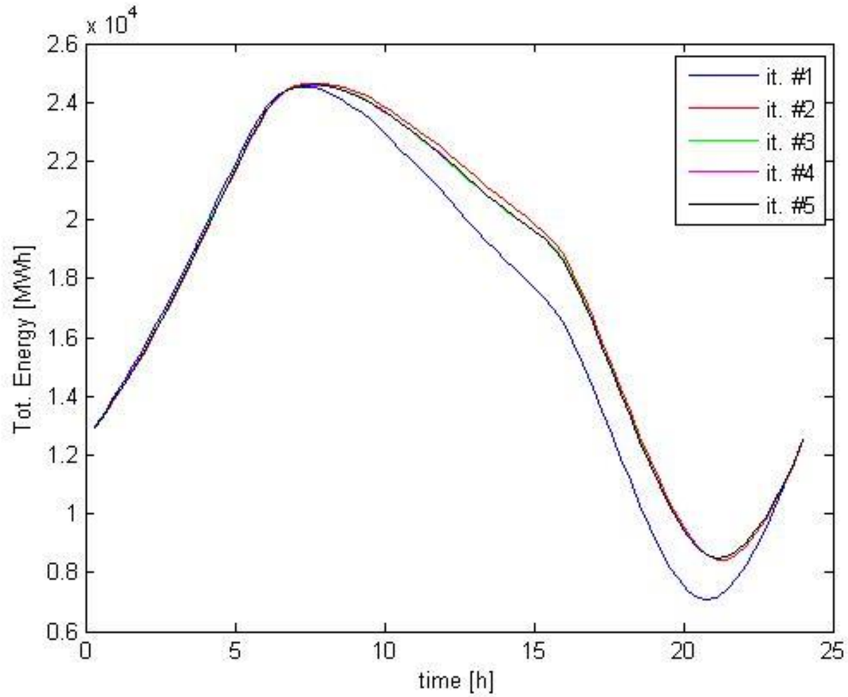
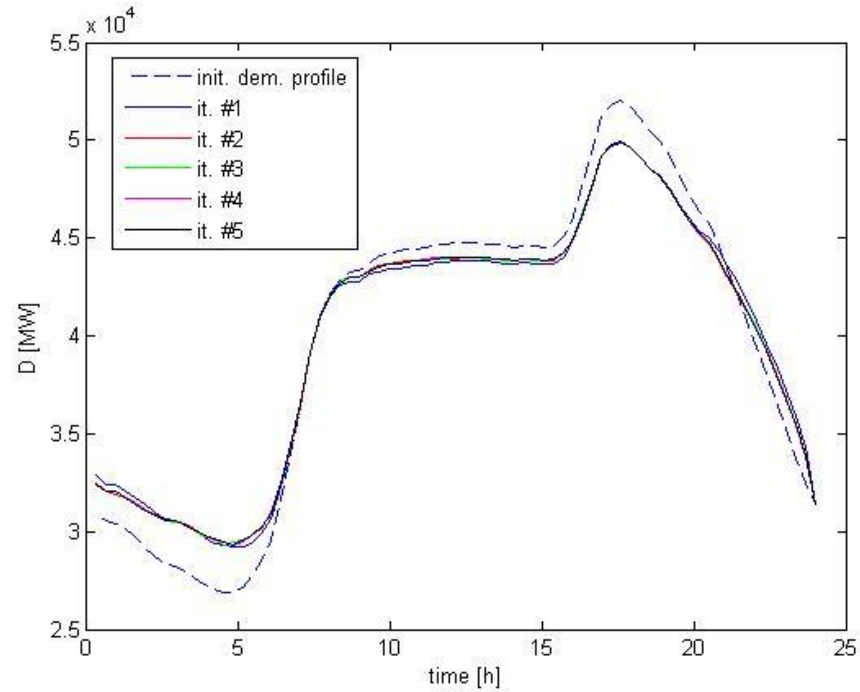
Simulations (3)





Total stored energy at different iterations of the HJB and transport equation

The new demand profiles are very similar to the ones obtained with the previous approach

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5. Devices with different parameters

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Different populations of devices (1)

<u>SO FAR</u>: all devices have the same parameters

<u>NEW APPROACH</u>: consider finite number N of populations, each with different parameters

HJB equation for the i-th population:

$$-\partial_t V_i(t,E_i,I_i) = \inf_{u_i}\left[p\left(D_o(t)+D_{ST}(t)\right)[u_i+\gamma_i u_i^2]+\partial_{E_i}V(t,E_i,I_i)u_i+\partial_{I_i}V(t,E_i,I_i)u_i\right]$$

Total demand variation introduced by storage

<u>NOTE</u>: the **only** interdependence between the HJBs of different populations is given by the price of energy $p(D_0(t)+D_{ST}(t))$

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Different populations of devices (2)

The fixed point at each t will be given by: $\left(p^*(t), u_1^*(t, E_1, I_1), \ldots, u_N^*(t, E_N, I_N)\right)$

Price if optimal control u* is applied:

$$p^*(t) = p\left(D_0(t) + \int_{E_1, I_1} m_1(t, E_1, I_i) u_1^*(t, E_1, I_i) + \ldots + \int_{E_N, I_N} m_N(t, E_N, I_N) u_N^*(t, E_N, I_N)\right)$$

Contribution to demand of the N-th population

Optimal control $u_i^*(t,E)$

$$u_i^*(t, E_i, I_i) = \arg\min_{u_i}\left[p^*(t)(u_i + \gamma u_i^2) + \partial_{E_i} V_i(t, E_i, I_i) u_i + \partial_{I_i} V_i(t, E_i, I_i) u_i\right]$$

At each time t the values of p and (u_1*,…,u_N*) are calculated iteratively until convergence is achieved*

➡

*The computational complexity is **linear** with respect to the number N of considered populations*

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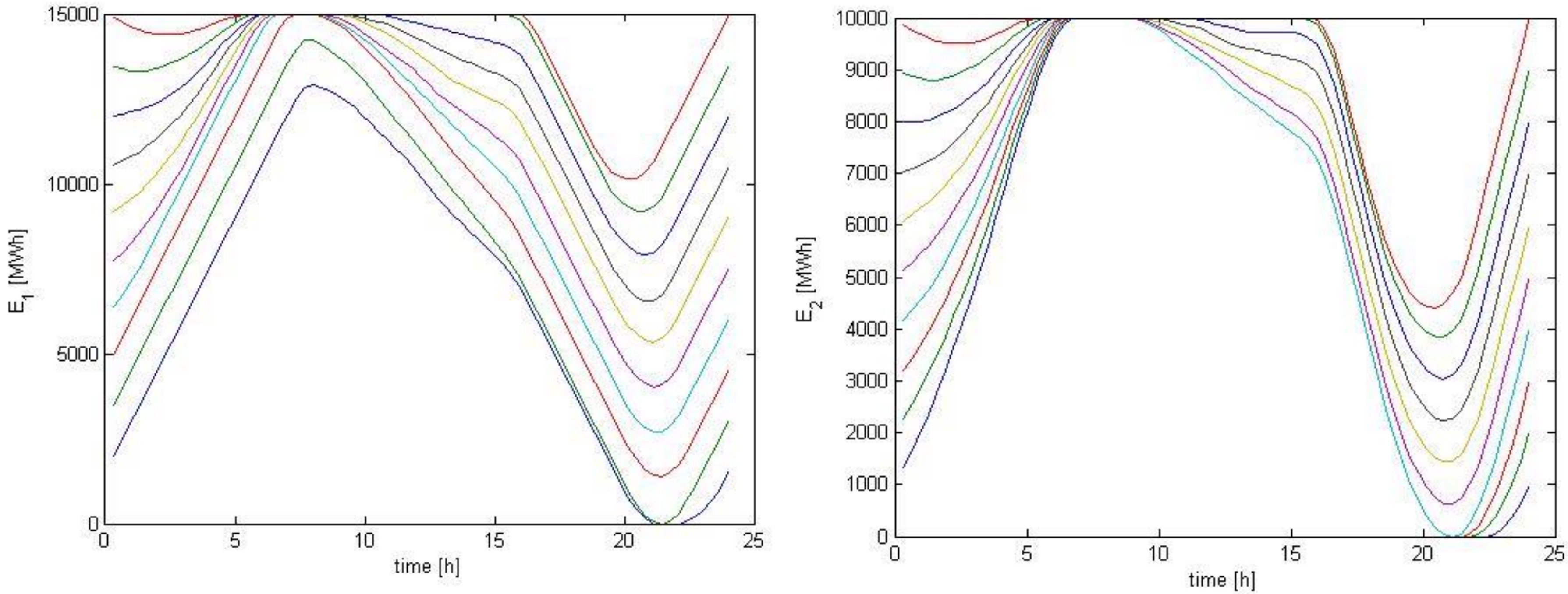
Simulations (1)

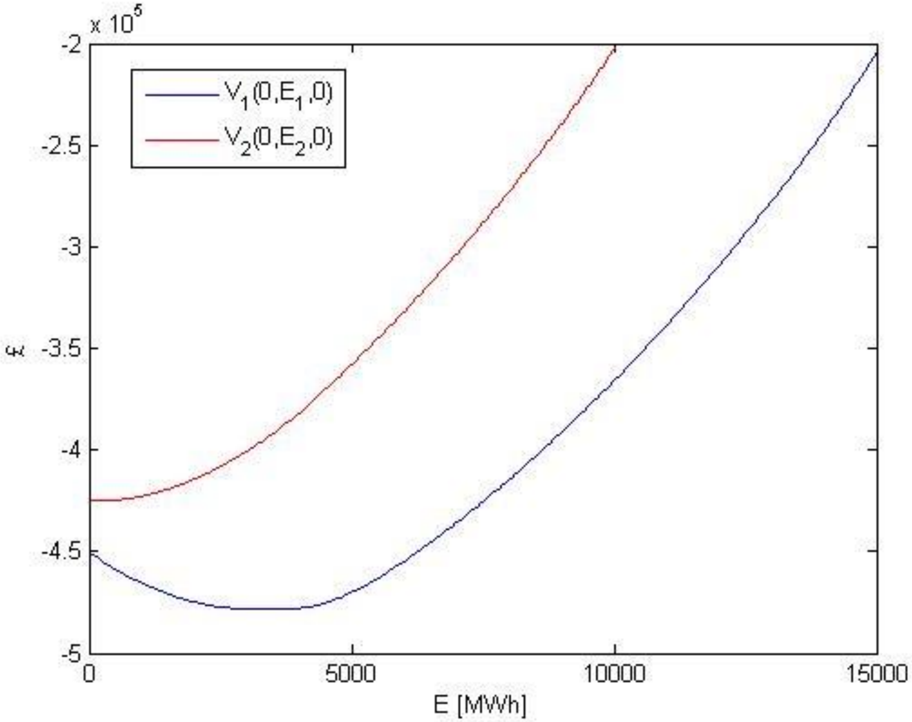
Parameters: $N = 2$ $E^1_{MAX}: 15GWh$ $u^1_{MAX} = -u^1_{MIN} = 1.5GW$ $\gamma_1 = 0.9 \cdot 10^{-4}$

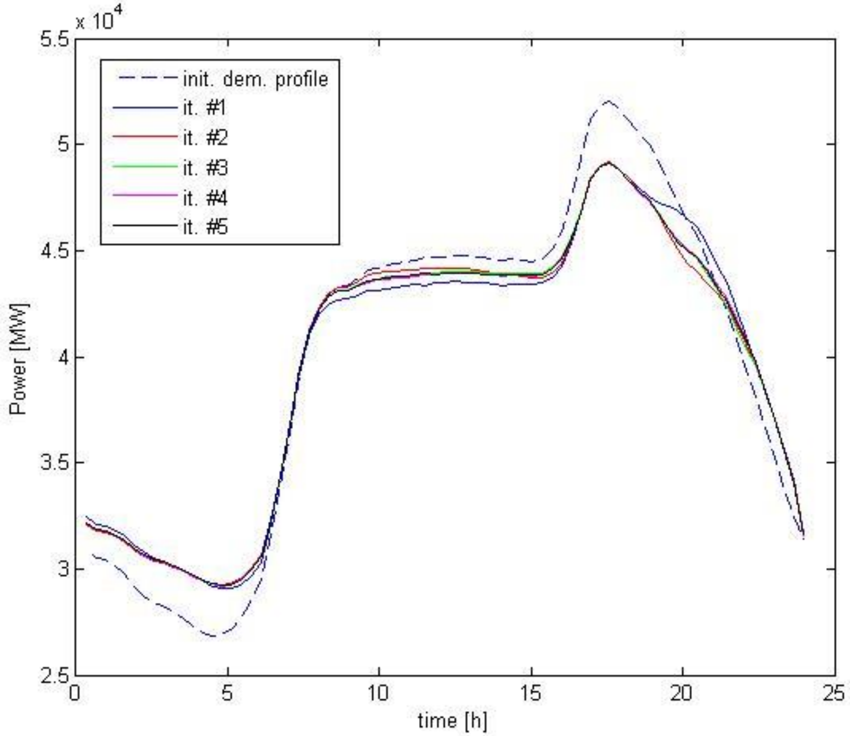
Number of populations $E^2_{MAX}: 10GWh$ $u^2_{MAX} = -u^2_{MIN} = 2GW$ $\gamma_2 = 1.1 \cdot 10^{-4}$



Energy levels of the storage devices with different initial energies for the two populations

Simulations (2)





Values of the optimal cost-to-go functions $V_i(0,E_i,0)$ for the two populations of devices

Demand profiles for different iterations of the HJB and transport equations

8. Arbitrage in multi-area systems

Introduction

SO FAR: topology of the network and transmission constraints are not taken into account

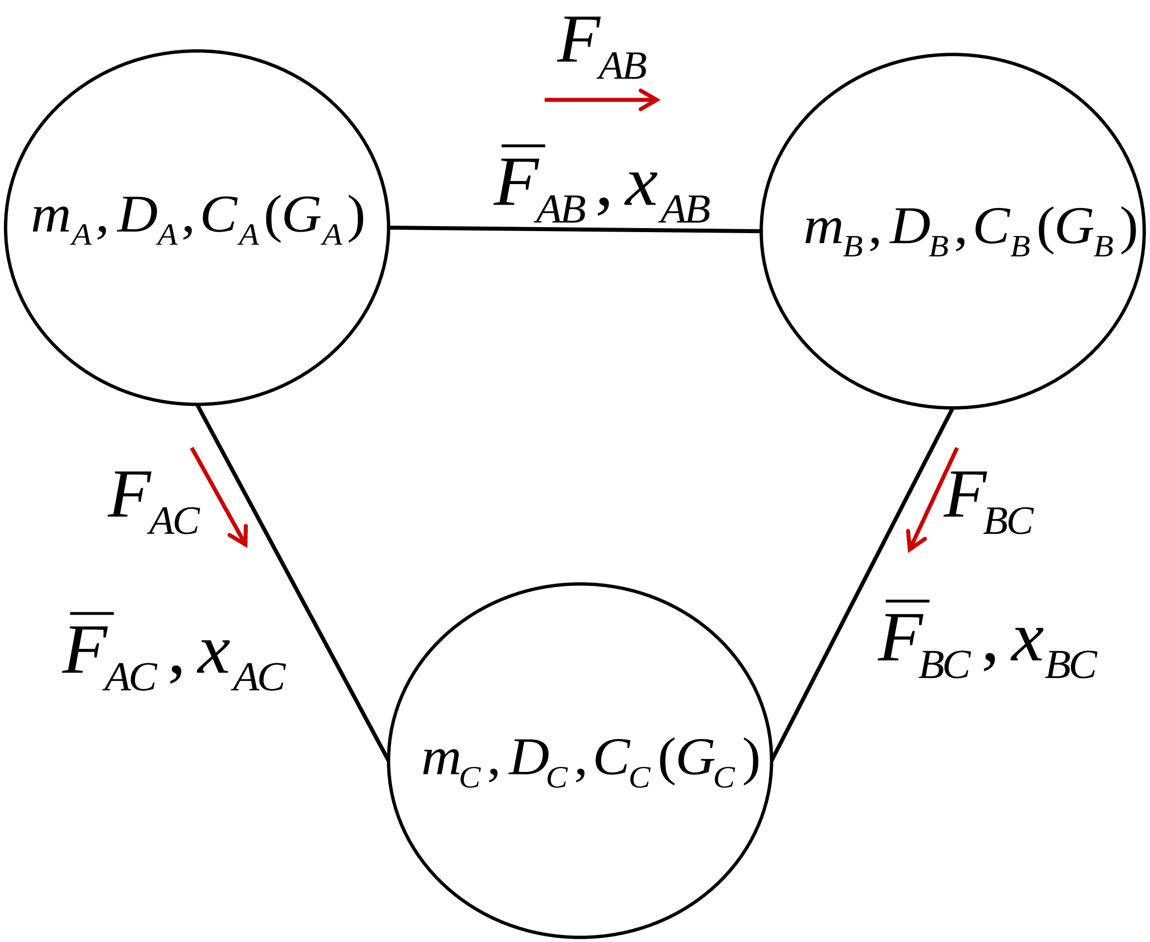
NOW: consider a system divided in several areas connected by transmission lines of **limited capacity**

↓

- Obtain the MFG equations for the populations of devices (one for each area), taking into account the new constraint
- Evaluate whether the convergence of the iterations to the optimal solution still holds for this new case

Three-area system

$m:$ Distribution of devices $D:$ Demand profile $C(G):$ Generation cost



$F:$ Power flow

$\overline{F}:$ Max capacity

$x:$ Line reactances

$G:$ Generated power

In simulations:

$\overline{F}_{AB} = 6GW \qquad x_{AB} = 0.2p.u.$

$\overline{F}_{BC} = 3.5GW \qquad x_{BC} = 0.1p.u.$

$\overline{F}_{AC} = 4.5GW \qquad x_{AC} = 0.2p.u.$

Economic dispatch

- Given demand and generation in each area, it is possible to calculate the resulting power flows F_{ij}:
  - voltage angles θ_{ij} are introduced and are considered small
  - transmission line resistances are ignored (*X* >>*R*)

- The generation cost $C_i(G_i)$ in the *i-th* area is chosen to be quadratic

WE ASSUME: $C_A(G) > C_B(G) > C_C(G)$ $\quad D_A > D_B > D_C$

- The economic dispatch is calculated by minimizing the total cost of supplying demand, solving a quadratic programming problem:

$$\min_{I,\theta} \sum_i C_i(G_i)$$

$$s.t. \quad G_i = D_i + I_i$$

$$I = Y\theta$$

$$F_{ij} = y_{ij}(\theta_i - \theta_j) \leq \overline{F}_{ij}$$

I_i : Net inflow in area *i*

Y : Admittance matrix

The price p_i at which devices in area i exchange energy is given by the Lagrange multiplier associated with the constraint:

$$I_i = Y\theta_i$$

MFG in three-area system

- The charge/discharge of storage devices is modelled as a variation of demand in the same area

$$D_i \rightarrow D_i + \int m_i(E_i)[u_i^*(E_i) + \gamma_i u_i^{*2}(E_i)]dE_i$$

- The HJB equations are very similar to the previous cases. For area A:

$$-\partial_t V_A(t,E_A) = \min_{u_A}\left[p_A^*(t)(u_A + \gamma_A u_A^2) + \partial_{E_A} V(t,E_A)u_A\right]$$

- The fixed point (p*(t),u*(t)) at each time step of the HJB is calculated with the following iterations (until convergence):

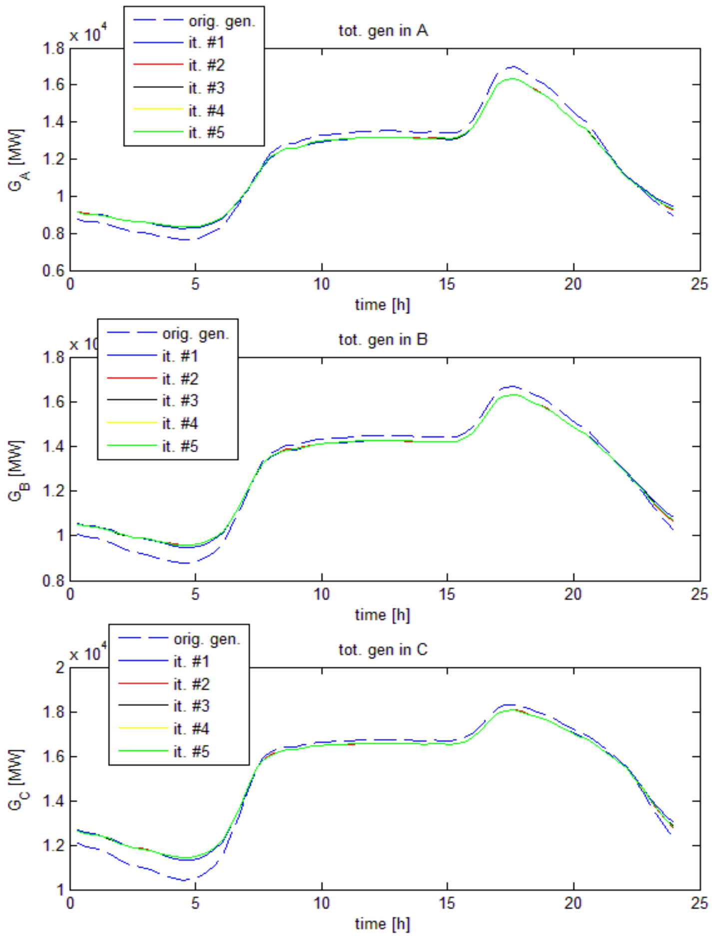
  1) given initial prices (p_A*, p_B*,p_C*), optimal controls (u_A*,u_B*, u_C*) are calculated

  2) the ***demand variation introduced by storage*** and the ***new flows*** are calculated

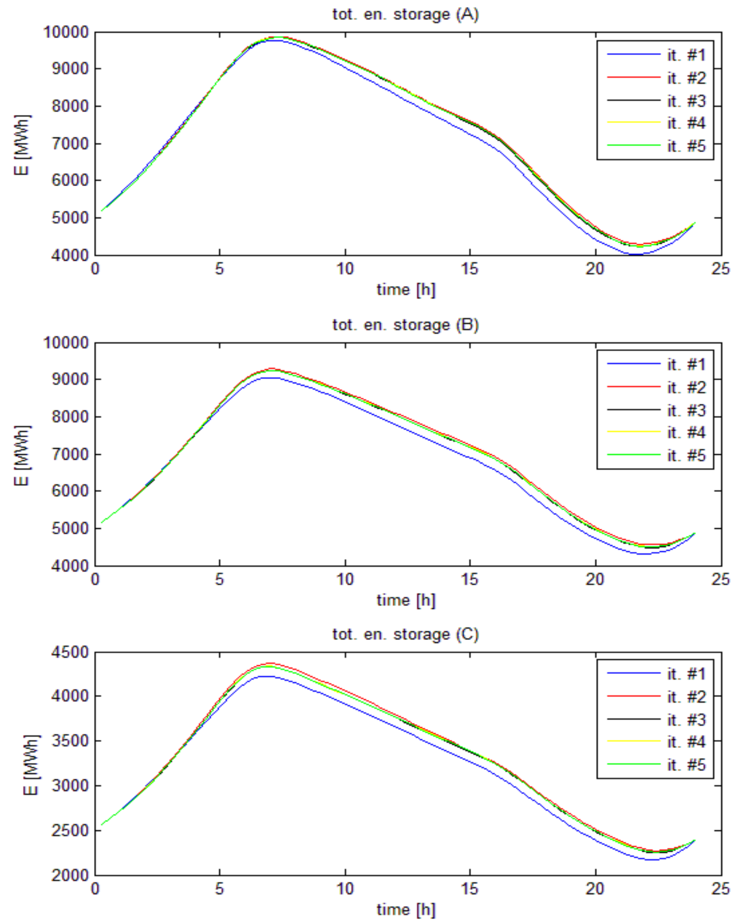
  3) A new economic dispatch is performed and the prices updated

Simulations (1)

Generation profiles

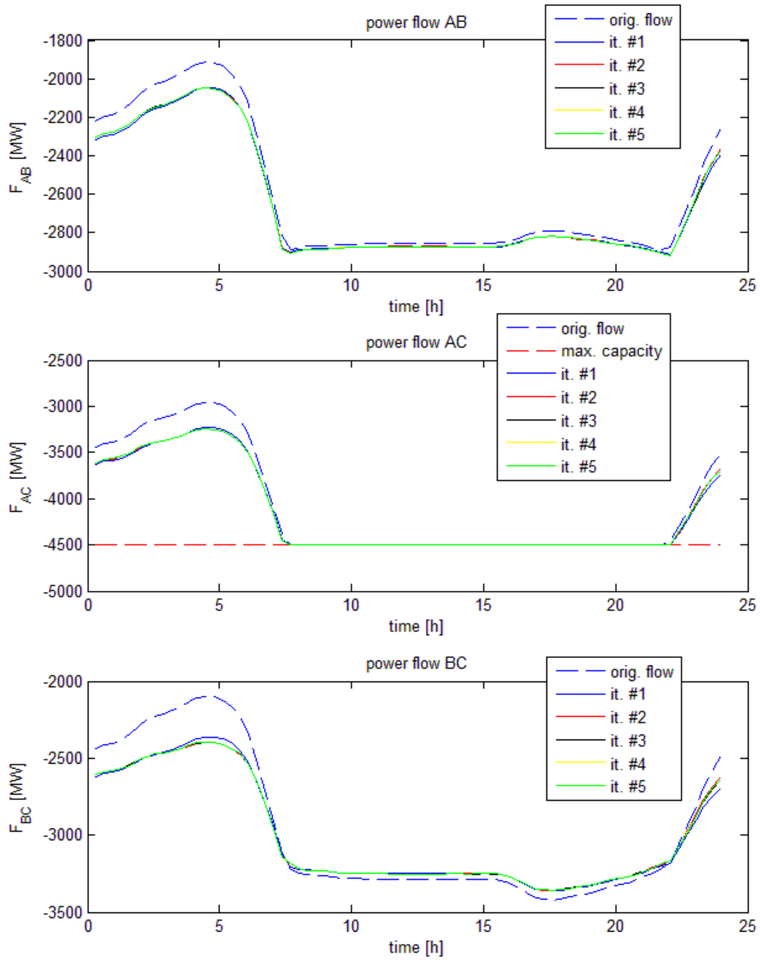


Total stored energy

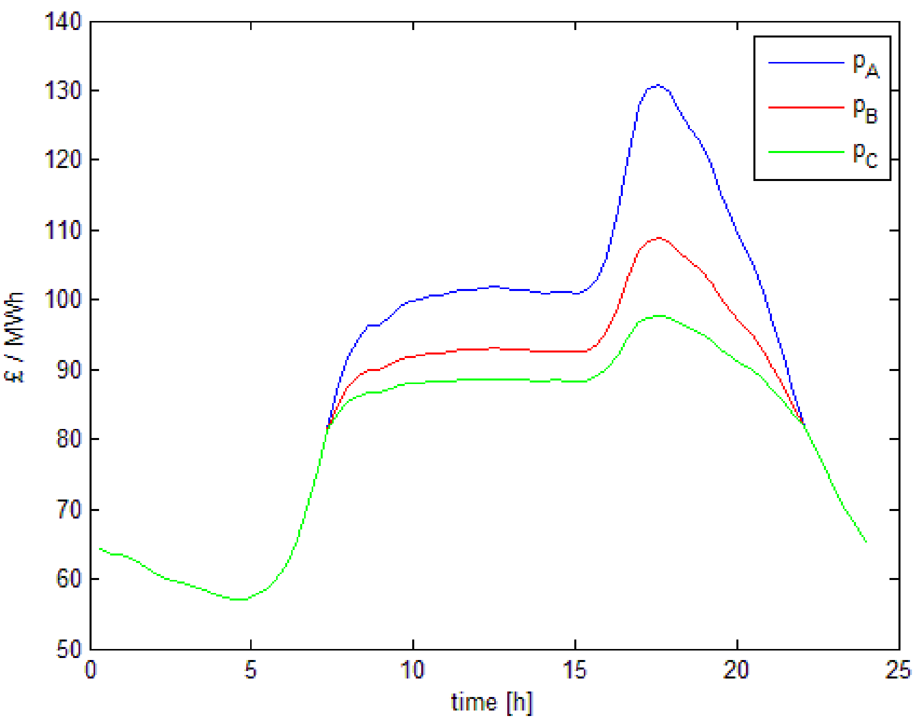


Simulations (2)

Power flows



Nodal prices



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Conclusions

- Modelling of dynamic electricity markets with large number of players

- Numerical integration schemes for finding Nash equilibria

- Extensions to allow for non-homogeneous players, and multi-area networks

- Open issues: existence of solutions and convergence of numeric schemes

- On-going work: characterization of Nash equilibria for unidirectional energy exchange between appliances and grid

THANK YOU