Roles of energy storage in balancing power networks

(Joint work with Richard Gibbens and James Cruise)

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Outline

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Hence there are major problems both of variability and of uncertainty.

Uses of storage

- Smoothing of variability — natural to address through arbitrage, buy energy when price is low, and sell energy when price is high.
- Buffering and control of uncertainty — can integrate with above by valuing energy in store.

Many other uses of storage, but we shall concentrate on the above two.

Dinorwig: capacity: 9 GWh rate: 1.8 GW efficiency 0.75–0.80

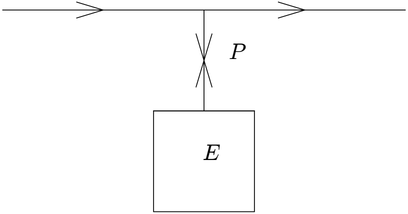
Outline

Storage for arbitrage

Assume (initially)

- The value of storage can be captured in price arbitrage (buy energy when cheap and sell when expensive)
- Prices are known in advance (no uncertainty)

Model

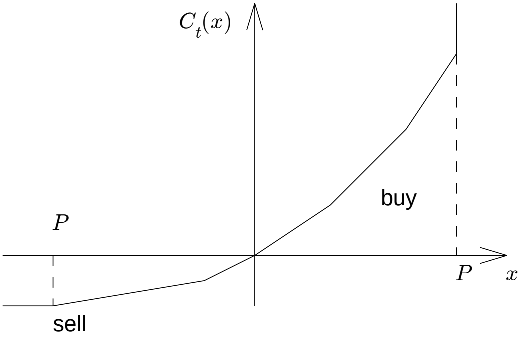


E = size of store — capacity constraint

P = max input/output rate — rate constraint

Cost function

At any (discrete) time t,



$C_t(x) =$ cost of increasing level of store by x (positive or negative)
Assume convex (reasonable). This may model

- market impact
- efficiency of store
- rate constraints

Problem

Let S_t = level of store at time t, $\qquad 0 \le t \le T$.

Policy $S = (S_0, \ldots, S_T)$, $\qquad S_0 = S_0^*$ (fixed), $\quad S_T = S_T^*$ (fixed).

Define also $x_t(S) = S_t - S_{t-1}$
(energy "bought" by store at time t – positive or negative)

Problem: minimise cost

$$\sum_{t=1}^{T} C_t(x_t(S))$$

subject to

$$S_0 = S_0^*, \qquad S_T = S_T^*$$

and

$$0 \le S_t \le E, \qquad 1 \le t \le T-1.$$

Result: Lagrangian sufficiency

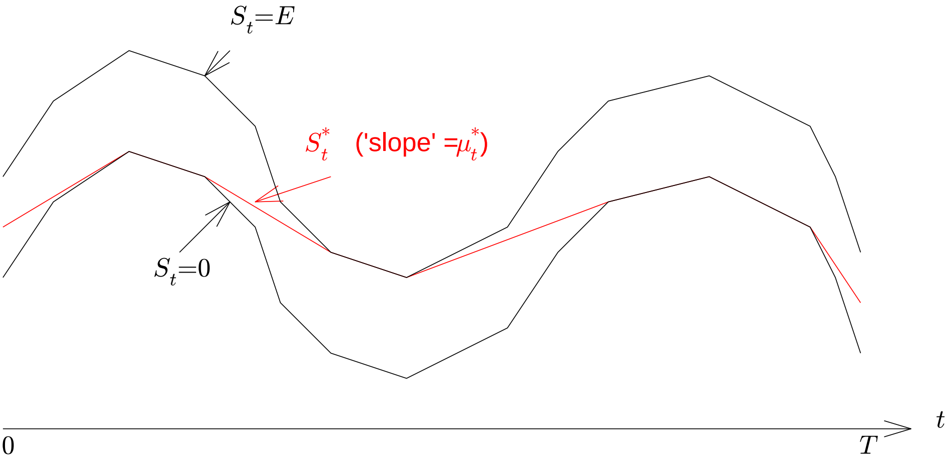
Suppose there exists a vector $\mu^* = (\mu_1^*, \ldots, \mu_T^*)$ and a value $S^* = (S_0^*, \ldots, S_T^*)$ of S such that

- S^* is feasible,
- $x_t(S^*)$ minimises $C_t(x) - \mu_t^* x$ over all x, $1 \leq t \leq T$,
- the pair (S^*, μ^*) satisfies the complementary slackness conditions, for $1 \leq t \leq T-1$,

$$\begin{cases} \mu_{t+1}^* = \mu_t^* & \text{if } 0 < S_t^* < E, \\ \mu_{t+1}^* \leq \mu_t^* & \text{if } S_t^* = 0, \\ \mu_{t+1}^* \geq \mu_t^* & \text{if } S_t^* = E. \end{cases} \tag{1}$$

Then S^* solves the stated problem.

Picture proof



Proper proof

Let S be any vector which is feasible for the problem (with $S_0 = S_0^*$ and $S_T = S_T^*$). Then, from the minimisation condition above,

$$\sum_{t=1}^{T} [C_t(x_t(S^*)) - \mu_t^* x_t(S^*)] \leq \sum_{t=1}^{T} [C_t(x_t(S)) - \mu_t^* x_t(S)] \,.$$

Rearranging and recalling that S and S^* agree at 0 and at T, we have

$$\begin{aligned}
\sum_{t=1}^{T} C_t(x_t(S^*)) - \sum_{t=1}^{T} C_t(x_t(S)) &\leq \sum_{t=1}^{T} \mu_t^*(S_t^* - S_{t-1}^* - S_t + S_{t-1}) \\
&= \sum_{t=1}^{T-1} (S_t^* - S_t)(\mu_t^* - \mu_{t+1}^*) \\
&\leq 0,
\end{aligned}$$

by the complementary slackness conditions above.

Comment

- The above result (essentially an application of the Lagrangian Sufficiency Principle) does not require convexity of the functions C_t.
- However, convexity the functions C_t is sufficient to guarantee the existence of a pair (S^*, μ^*) as above.

  The latter result is a standard application of Strong Lagrangian theory (i.e. the Supporting Hyperplane Theorem).

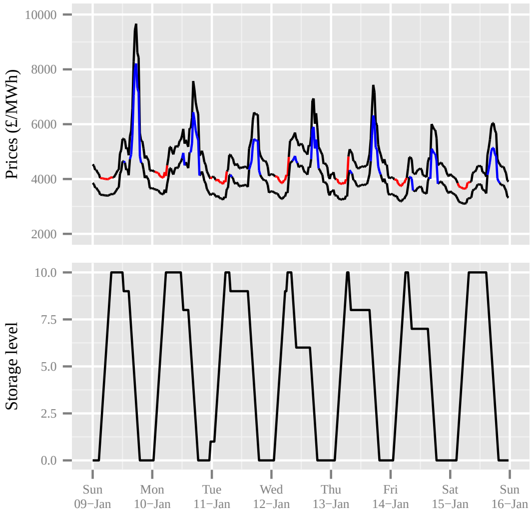
Algorithm

- Need to identify the relevant value μ_t^* at each time t.
- This is as in the previous picture ("stretched string").
- Note that, at each time t, only a typically short time horizon is required to identify μ_t^*.

Example: Real prices with Dinorwig parameters

E/P = 5 hrs   Efficiency = 0.85 (ratio of sell to buy price).

Solution is bang-bang: red points buy, blue points sell



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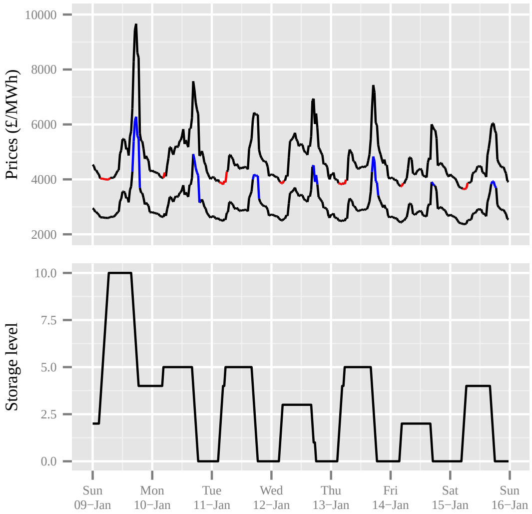

Small store

This is a store whose activities are not so great as to impact upon the market, and which thus has linear buy and sell prices. Thus, for all t,

$$C_t(x) = \begin{cases} c_t^{(b)} x & \text{if } 0 \leq x \leq P \\ c_t^{(s)} x & \text{if } -P \leq x < 0 \end{cases}$$

where $0 < c_t^{(s)} \leq c_t^{(b)}$ and P is rate constraint.

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Characteristics

- Optimal control is bang-bang: at each time buy as much as possible, do nothing, or sell as much as possible (according to value of μ_t^* in relation to above costs).
- If $E = \infty$ (no capacity constraint) then optimal solution is global in time.
- If $P = \infty$ (no rate constraint) then optimal solution is very local in time.

Outline

Result

Suppose that the cost functions C_t evolve randomly in time, and that

$$C_t = \xi_t \bar{C}_t, \qquad 1 \le t \le T,$$

where $(\bar{C}_1, \dots, \bar{C}_T)$ is a sequence of deterministic cost functions and where $(\xi_1, \dots, \xi_T)$ is a sequence of strictly positive real-valued random variables such that

$$\mathbf{E}(\xi_t \,|\, \mathcal{F}_{t-1}) = \xi_{t-1}, \qquad 1 \le t \le T,$$

and where the deterministic functions $\bar{C}_t$ are assumed to satisfy the same conditions as the cost functions C_t of the deterministic problem.
Then the optimal control (cost minimisation) strategy remains exactly as previously, with the optimal sequence of store levels as given in the case where stochastic cost functions C_t are replaced by their deterministic counterparts $\bar{C}_t$.

Outline

Integration of buffering for uncertainty

Suppose that the store is also used to provide buffering against relatively rare unexpected events.

Then the previous optimization problem may be rewritten as:
Problem: minimise

$$\sum_{t=1}^{T} \left[C_t(x_t(S)) + A_t(S_t) \right]$$

subject to

$$S_0 = S_0^*, \qquad S_T = S_T^*$$

and

$$0 \leq S_t \leq E, \qquad 1 \leq t \leq T - 1.$$

Here the functions A_t, such that $A_t(S_t)$ is the cost (negative benefit) of having a level of reserve S_t in the store at time t, are independently calculable.

Result - Lagrangian sufficiency

Assume, for simplicity, we incorporate the capacity constraints into the functions A_t, and the functions C_t and A_t are differentiable.

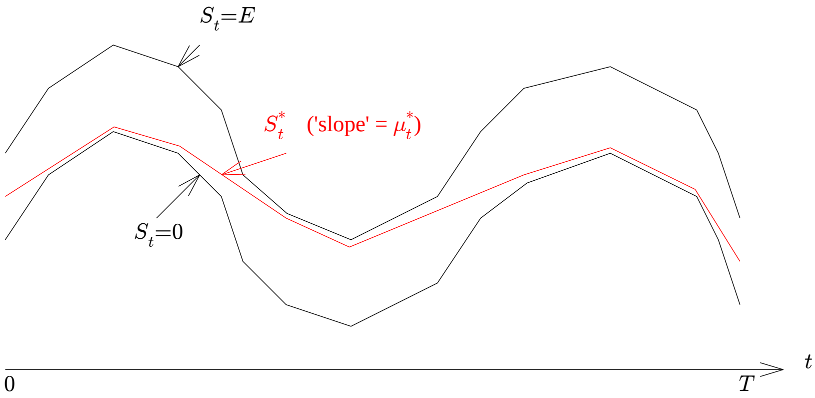
Suppose there exists a scalar ν^* and a value $S^* = (S_0^*, \ldots, S_T^*)$ (with S_0^* and S_T^* having their required values) such that

$$C_t'(x_t) = \nu - \sum_{u=t}^{T} A_u'(S_u) \quad =: \mu_t$$

Then S^* solves the stated problem.

Lagrangian sufficiency (ctd)

The solution has a "stretched string" characterisation as before:



The previous algorithm may be adapted to give a solution which is essentially local in time.

Outline

Demand-side management

Like storage, this is shifting energy through time.

However, there are typically additional constraints e.g.

- duration of actions
- frequency of actions.