20th British Combinatorial Conference

University of Durham
(organized jointly with The Open University)

10th to 15th July 2005

Programme

Contents

The week at a glance

Sunday: 14:00–21:00 Arrival and registration in Collingwood College
(Conference office in Room CM103 from Monday)
19:00–21:00 Dinner in Collingwood

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
07:45	Breakfast in Collingwood	Breakfast in Collingwood	Breakfast in Collingwood	Breakfast in Collingwood	Breakfast in Collingwood
09:00	**WELCOME**				
09:15	**Sokal** CG93	**Seymour** CG93	**Scott** CG93	**Östergård** CG93	**Steger** CG93
10:15	Refreshments	Refreshments	Refreshments	Refreshments	Refreshments
10:45	Contributed talks (4 slots)	Contributed talks (4 slots)	Contributed talks (4 slots)	Contributed talks (4 slots)	Contributed talks (4 slots)
12:30	Lunch in Collingwood	Lunch in Collingwood	Lunch in Collingwood	Lunch in Collingwood	Lunch in Collingwood
13:30			Excursion to Beamish Museum	Editorial meeting	
14:00	Contributed talks (5 slots)	Contributed talks (5 slots)		Contributed talks (3 slots)	Contributed talks (2 slots)
15.00					**Green** CG93
15:15				Problem session CG93	
16:00	Refreshments	Refreshments		Refreshments	Refreshments
16:30	**King** CG93	**Serra** CG93		**Penttila** CG93	**END**
17.45		Business meeting CG93			
18:30	Reception in Collingwood JCR			Reception in Collingwood JCR	
19:00		Dinner in Collingwood	Dinner in Collingwood	**CONFERENCE DINNER** Collingwood Dining Hall	
19:15	Dinner in Collingwood				
20:15		Durham walking tour			
20:30	ICA meeting		BCC committee meeting		

The programme in detail

Sunday

14:00–21:00	Arrival and registration in Collingwood College (Conference office in Room CM103 from Monday)
19:00–21:00	Dinner in Collingwood

Monday

07:45–08:30	Breakfast in Collingwood
09:00–09:10	Welcome CG93 Sir Kenneth Calman, Vice Chancellor
09:15–10:15	Alan Sokal (Chair: Mark Jerrum) CG93 *The multivariate Tutte polynomial (alias Potts model) for graphs and matroids*
10:15–10:45	Refreshments
10:45–12:20	Contributed talks (4 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
12:30–13:30	Lunch in Collingwood
14:00–16:00	Contributed talks (5 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
16:00–16:25	Refreshments
16:30–17:30	Oliver King (Chair: Bridget Webb) CG93 *The subgroup structure of finite classical groups in terms of geometric configurations*
18:30–19.00	Reception in Collingwood JCR
19:15–20:15	Dinner in Collingwood
20:30	ICA meeting

Monday morning contributed talks

CG93

10:45–11:05	**M. Johnson**	Connectedness of graphs of vertex-colourings
11:10–11:30	**L. Cereceda**	Connectedness of graphs of 3-colourings
11:35–11:55	**P. Wang**	The equitable colouring of plane graphs with large girth
12:00–12:20	**R. Häggvist**	A $\Delta + 4$ bound on the total chromatic number for graphs with chromatic number on the order of $\sqrt{\Delta / \log \Delta}$

CG60

10:45–11:05	**M. Cera**	Average degree and extremal problems for infinite graph
11:10–11:30	**P. García-Vazquez**	Optimal restricted connectivity and superconnectivity in graphs with small diameter
11:35–11:55	**X. Marcote**	On the connectivity of a product of graphs
12:00–12:20	**J.C. Valenzuela**	New results on the Zarankiewicz problem

CG83

10:45–11:05	**D.H. Smith**	Cyclically permutable codes and simplex codes
11:10–11:30	**S.K. Houghten**	Bounds on optimal edit metric codes
11:35–11:55	**T. Maruta**	On optimal non-projective ternary linear codes
12:00–12:20	**M. Shinohara**	Constructing linear codes from some orbits of projectivities

CG85

10:45–11:05	**A. Yeo** Total domination in graphs
11:10–11:30	**M.D. Plummer** Domination in a graph with a 2-factor
11:35–11:55	**V.E. Zverovich** A generalised upper bound for the k-tuple domination number
12:00–12:20	**D. Mojdeh** Domination number of some 3-regular graphs

CG232

10:45–11:05	**A. de Mier** The lattice of cycle flats of a matroid
11:10–11:30	**M. Jerrum** Two remarks concerning balanced matroids
11:35–11:55	**C.J. Colbourn** Covering Arrays of Strength Two
12:00–12.20	**R.A. Walker II** Tabu search for Covering Arrays using permutation vectors

Summary of Monday morning speakers

	CG93	CG60	CG83	CG85	CG232
10:45–11:05	M. Johnson	Cera	Smith	Yeo	de Mier
11:10–11:30	Cereceda	García-Vazquez	Houghten	Plummer	Jerrum
11:35–11:55	Wang	Marcote	Maruta	Zverovich	Colbourn
12:00–12:20	Häggvist	Valenzuela	Shinohara	Mojdeh	Walker II

Monday afternoon contributed talks

CG93

14:00–14:20	**S. Ball**	
	A new approach to finite semifields	
14:25–14:45	**A. Cossidente**	
	Ovoids of the Hermitian surface and derivations	
14:50–15:10	**G. Marino**	
	Special sets of the Hermitian surface and Segre invariants	
15:15–15:35	**R. Shaw**	
	Grassmann and Segre varieties over GF(2): some graph theory links	
15:40–16:00	**T.L. Alderson**	
	Optical orthogonal codes: new constructions	

CG60

14:00–14:20	**K. Cameron**	
	Coflow and covering vertices by directed circuits	
14:25–14:45	**K. Mynhardt**	
	Maximal increasing paths in edge-ordered trees	
14:50–15:10	**Y. Egawa**	
	Existence of disjoint cycles containing specified vertices	
15:15–15:35	**K. Yoshimoto**	
	The number of cycles in 2-factors of line graphs	
15:40–16:00	**J. Fujisawa**	
	Long cycles passing through a linear forest	

CG83

14:00–14:20	**D. Kahrobaei**	
	A graphic generalisation of Arithmetic	
14:25–14:45	**M. Tsuchiya**	
	Chordal double bound graphs and posets	
14:50–15:10	**A. Lev**	
	Bertrand Postulate, the Prime Number Theorem and product anti-magic graphs	
15:15–15:35	**H. Fernau**	
	A sum labelling for the flower $f_{q,p}$	
15:40–16:00	**C. Balbuena**	
	Consecutive magic graphs	

CG85

14:00–14:20	**B.S. Webb** Representing $(d,3)$-tessellations as quotients of Cayley maps
14:25–14:45	**A.V. Gagarin** Structure and enumeration of toroidal and projective-planar graphs with no $K_{3,3}$'s
14:50–15:10	**M. Šajna** Self-complementary two-graphs and almost self-complementary double covers over complete graphs
15:15–15:35	**G. Mazzuoccolo** Doubly transitivity on 2-factors
15:40–16:00	**E.V. Konstantinova** Reconstruction of permutations from their erroneous patterns

CG232

14:00–14.20	**P.E. Chigbu** Admissible permutations for constructing Trojan squares for $2n$ treatments with odd-prime n side
14:25–14.45	**A. Drápal** surgeries on latin trades
14:50–15:10	**A.D. Keedwell** A new criterion for a Latin square to be group-based
15:15–15:35	**L.-D. Öhman** The intricacy of avoiding arrays
15:40–16.00	**N. Cavenagh** A superlinear lower bound for the size of a critical set in a latin square

Summary of Monday afternoon speakers

	CG93	CG60	CG83	CG85	CG232
14:00–14:20	Ball	K. Cameron	Kahrobaei	Webb	Chigbu
14:25–14:45	Cossidente	Mynhardt	Tsuchiya	Gagarin	Drápal
14:50–15:10	Marino	Egawa	Lev	Šajna	Keedwell
15:15–15:35	Shaw	Yoshimoto	Fernau	Mazzuoccolo	Öhman
15:40–16:00	Alderson	Fujisawa	Balbuena	Konstantinova	Cavenagh

Tuesday

07:45–08:30	Breakfast in Collingwood
09:15–10:15	Paul Seymour (Chair: Nigel Martin) CG93 *The structure of claw-free graphs*
10:15–10:45	Refreshments
10:45–12:20	Contributed talks (4 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
12:30–13:30	Lunch in Collingwood
14:00–16:00	Contributed talks (5 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
16:00–16:25	Refreshments
16:30–17:30	Oriel Serra (Chair: Peter Rowlinson) CG93 *An isoperimetric method for the small subset problem*
17.45–18.30	Business meeting in CG93
19:00–20:00	Dinner in Collingwood
20:15	Durham walking tour

Tuesday morning contributed talks

CG93

10:45–11:05 **J.E. Dunbar**
One small step towards proving the PPC

11:10–11:30 **M. Frick**
A new perspective on the Path Partition Conjecture

11:35–11:55 **K.L. McAvaney**
The Path Partition Conjecture

12:00–12:20 **D.A. Pike**
Pancyclic PBD block-intersection graphs

CG60

10:45–11:05 **P. Butkovič**
Max-algebra: the linear algebra of combinatorics?

11:10–11:30 **M. Giudici**
All vertex-transitive locally-quasiprimitive graphs have a semiregular automorphism

11:35–11:55 **A. Miralles**
On the Frobenius problem of three numbers: Part I

12:00–12:20 **F. Aguiló**
On the Frobenius problem of three numbers: Part II

CG83

10:45–11:05 **F. Benmakrouha**
Validation of a particular class of bilinear systems

11:10–11:30 **P. Lisoněk**
Combinatorial families enumerated by quasi-polynomials

11:35–11:55 **R. Johnson**
Universal cycles for permutations and other combinatorial families

12:00–12:20 **M. Nakamura**
Broken circuits and NBC complexes of convex geometries

CG85

10:45–11:05	**M. Luz Puertas** On the metric dimension of graph products
11:10–11:30	**O. Oellermann** The strong metric dimension of graphs
11:35–11:55	**P. Dankelmann** Distance and Inverse Degree
12:00–12:20	**C. Seara** On monophonic sets in graphs

CG232

10:45–11:05	**L. Gionfriddo** Hexagon Biquadrangle systems
11:10–11:30	**S. Küçükçifçi** Maximum packings for perfect four-triple configurations
11:35–11:55	**E.J. Billington** Equipartite and almost-equipartite gregarious 4-cycle systems
12:00–12:20	**K. Ushio** Balanced C_4-quatrefoil designs

Summary of Tuesday morning speakers

	CG93	CG60	CG83	CG85	CG232
10:45–11:05	Dunbar	Butkovič	Benmakrouha	Luz Puertas	Gionfriddo
11:10–11:30	Frick	Giudici	Lisoněk	Oellermann	Küçükçifçi
11:35–11:55	McAvaney	Miralles	R. Johnson	Dankelmann	Billington
12:00–12:20	Pike	Aguiló	Nakamura	Seara	Ushio

Tuesday afternoon contributed talks

CG93

14:00–14:20	**A.J.W. Hilton**	$(r, r+1)$-factorizations of multigraphs with high minimum degree
14:25–14:45	**R.J. Waters**	Some list colouring problems in the reals
14:50–15:10	**F.C. Holroyd**	Multiple chromatic numbers of some Kneser graphs
15:15–15:35	**M. Škoviera**	Factorisation of snarks
15:40–16:00	**E. Máčajová**	On the strong circular 5-flow conjecture

CG60

14:00–14:20	**M. Liazi**	Polynomial variants of the densest/heaviest k-subgraph problem
14:25–14:45	**K. Vušković**	Combinatorial algorithm for finding a clique of maximum weight in a C_4-free Berge graph
14:50–15:10	**S. Zenia**	Quasi-locally $P^*(\omega)$ graphs
15:15–15:35	**H. Ait Haddadène**	Perfect graphs and vertex colouring problem of a graph
15:40–16:00	**B. Yalaoui**	On related combinatory problems in information cartography

CG83

14:00–14:20	**C. Elsholtz**	Maximal sets of unit-distance points
14:25–14:45	**C.H. Cooke**	Bounds on element order in rings Z_m with divisors of zero
14:50–15:10	**S. Bouroubi**	Bell's number in the Alekseev inequality
15:15–15:35	**I-C. Huang**	Variable changes in generalized power series
15:40–16:00	**C.G. Rutherford**	Coprime polynomials over $GF(2)$

CG85

14:00–14:20	**Z. Radosavljević** On bicyclic reflexive graphs
14:25–14:45	**D. Cvetković** Signless Laplacians and line graphs
14:50–15:10	**S.K. Simić** Some new results on the index of trees
15:15–15:35	**F. Bell** On graphs with least eigenvalue -2
15:40–16:00	**P. Rowlinson** Independent sets in extremal strongly regular graphs

CG232

14:00–14:20	**E. Ş. Yazici** Minimal homogeneous Steiner triple trades
14:25–14:45	**A.P. Street** Defining sets of full designs and other simple designs
14:50–15:10	**J.C. Bate** Group Key distribution Patterns
15:15–15:35	**S. Huczynska** Frequency Permutation Arrays
15:40–16.00	**M. Sawa** An additive structure of BIB designs

Summary of Tuesday afternoon speakers

	CG93	CG60	CG83	CG85	CG232
14:00–14:20	Hilton	Liazi	Elsholtz	Radosavljević	Yazici
14:25–14:45	Waters	Vušković	Cooke	Cvetković	Street
14:50–15:10	Holroyd	Zenia	Bouroubi	Simić	Bate
15:15–15:35	Škoviera	Ait Haddadène	I-C. Huang	Bell	Huczynska
15:40–16:00	Máčajová	Yalaoui	Rutherford	Rowlinson	Sawa

Wednesday

07:45–08:30	Breakfast in Collingwood
09:15–10:15	Alex Scott (Chair: Graham Brightwell) CG93 The Rado Lecture *Judicious partitions and related problems*
10:15–10:45	Refreshments
10:45–12:20	Contributed talks (4 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
12:30–13:30	Lunch in Collingwood
13:30–18.30	Excursion to Beamish Museum
19:00–20:00	Dinner in Collingwood
20:30	BCC committee meeting

Wednesday morning contributed talks

CG93

10:45–11:05	**A. Berrachedi** Cycle regularity and Hypercubes	
11:10–11:30	**S. Ouatiki** On the domatic number of the 2-section graph of the order-interval hypergraph of a finite poset	
11:35–11:55	**P.-G. Tsikouras** Dominating sequences and traversals of ordered trees	
12:00–12:20	**H. Matsumura** On spanning trees with degree restrictions	

CG60

10:45–11:05	**O. Pikhurko** Fragmentability of bounded degree graphs	
11:10–11:30	**J. Wojciechowski** Edge-bandwidth of grids and tori	
11:35–11:55	**B. Zmazek** Retract-rigid strong graph bundles	
12:00–12:20	**J. Žerovnik** Hypercubes are distance graphs	

CG83

10:45–11:05	**R.F. Bailey** Permutation groups, error-correcting codes and uncoverings	
11:10–11:30	**J. Moori** Codes, Designs and Graphs from Finite Simple Groups	
11:35–11:55	**M.J. Grannell** A flaw in the use of minimal defining sets for secret sharing schemes	
12:00–12:20	**U. Grimm** On the number of power-free words in two and three letters	

CG85

10:45–11:05	**H.J. Broersma** Matchings, Tutte sets, and independent sets
11:10–11:30	**G. Rinaldi** One-factorizations of the complete graph with a prescribed automorphism group
11:35–11:55	**N.E. Clarke** The ultimate isometric number of a graph
12:00–12:20	**D.F. Manlove** "Almost stable" matchings in the Roommates problem

CG232

10:45–11:05	**Q. Kang** More large sets of resolvable MTS and DTS
11:10–11:30	**W-C. Huang** The Doyen-Wilson Theorem for Extended Directed Triple systems
11:35–11:55	**J. Arhin** On the structure of equireplicate partial linear spaces with constant line size
12:00–12:20	**A. Vietri** Difference families from infinite translation designs

Summary of Wednesday morning speakers

	CG93	CG60	CG83	CG85	CG232
10:45–11:05	Berrachedi	Pikhurko	Bailey	Broersma	Kang
11:10–11:30	Ouatiki	Wojciechowski	Moori	Rinaldi	W-C. Huang
11:35–11:55	Tsikouras	Zmazek	Grannell	Clarke	Arhin
12:00–12:20	Matsumura	Žerovnik	Grimm	Manlove	Vietri

Thursday

07:45–08:30	Breakfast in Collingwood
09:15–10:15	Patric Östergard (Chair: Stephanie Perkins) CG93 *Constructing combinatorial objects via cliques*
10:15–10:45	Refreshments
10:45–12:20	Contributed talks (4 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
12:30–13:30	Lunch in Collingwood
13:30–14:30	Editorial meeting
14:00–15:10	Contributed talks (3 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
15:15–16.00	Problem session in CG93
16:00–16:25	Refreshments
16:30–17:30	Tim Penttila (Chair: Simeon Ball) *Flocks of circle planes*
18:30–19:00	Reception in Collingwood JCR
19:00	Conference Dinner in Collingwood Dining Hall

Thursday morning contributed talks

CG93

10:45–11:05	**G. Sabidussi**	Deletion-similarity versus similarity of edges in graphs with few edge-orbits
11:10–11:30	**M. Priesler**	Partitioning a graph into two pieces each isomorphic to the other or to its complement
11:35–11:55	**H.C. Swart**	Minimal claw-free graphs
12:00–12:20	**I.A. Vakula**	claw-free graphs with non-clique μ-subgraphs and related geometries

CG60

10:45–11:05	**M.G. Parker**	Graph equivalence from equivalent quantum states
11:10–11:30	**A. Mohammadian**	On the zero-divisor graph of a ring
11:35–11:55	**N. Lichiardopol**	Cycles in a touirnament with pairwise zero, one or two given common vertices
12:00–12:20	**R. Tsaur**	Contractible digraphs, fixed cliques and the Cop-robber games

CG83

10:45–11:05	**A.C. Burgess**	Colouring even cycle systems
11:10–11:30	**I. Anderson**	A general approach to constructing power-sequence terraces for $\mathbb{Z}_n$
11:35–11:55	**L. Ellison**	Logarithmic terraces
12:00–12:20	**D.A. Preece**	Some $\mathbb{Z}_{n+2}$ terraces from $\mathbb{Z}_n$ power-sequences, n being an odd prime power

CG85

10:45–11:05 **E.L.C. King**
Comparing subclasses of well-covered graphs

11:10–11:30 **C.A. Whitehead**
Minimum dominating walks on graphs with large circumference

11:35–11:55 **E. Prisner**
k-pseudosnakes in n-dimensional hypercubes

12:00–12:20 **A. Finbow**
On well-covered planar triangulations

CG232

10:45–11:05 **L.A. Goldberg**
Approximate counting: Independent sets and Ferromagnetic Ising

11:10–11:30 **V. Grout**
Initial results from a study of probability curves for shortest arcs in optimal ATSP tours with application to heuristic performance

11:35–11:55 **N. Zagaglia-Salvi**
On very sparse circulant (0,1) matrices

12:00–12:20 **A. Alipour**
Negative Hadamard Graphs

Summary of Thursday morning speakers

	CG93	CG60	CG83	CG85	CG232
10:45–11:05	Sabidussi	Parker	Burgess	King	Goldberg
11:10–11:30	Priesler	Mohammadian	Anderson	Whitehead	Grout
11:35–11:55	Swart	Lichiardopol	Ellison	Prisner	Zagaglia-Salvi
12:00–12:20	Vakula	Tsaur	Preece	Finbow	Alipour

Thursday afternoon contributed talks

CG93

14:00–14:20	**S. Brandt**	Triangle-free graphs whose independence number equals the degree
14:25–14:45	**T. Kaiser**	The circular chromatic index of graphs of high girth
14:50–15:10	**D. Paulusma**	The computational complexity of the parallel knock-out problem

CG60

14:00–14:20	**B. Montágh**	New bounds on some Turán numbers for infinitely many n
14:25–14:45	**P. Borg**	Graphs with the Erdös-Ko-Rado property
14:50–15:10	**A. Abbas**	A family of large chordal ring of degree six

CG83

14:00–14:20	**F.E.S. Bullock**	Connected, nontraceable detour graphs
14:25–14:45	**J.E. Singleton**	Maximal nontraceable graphs of small size
14:50–15:10	**S.A. van Aardt**	Maximal non-traceable oriented graphs

CG85

14:00–14:20 **M. Abreu**
Graphs and digraphs with all 2-factors isomorphic

14:25–14:45 **D. Labbate**
Pseudo 2-factor isomorphic regular bipartite graphs

14:50–15:10 **N. Martin**
Unbalanced $K_{p,q}$ factorisations of complete bipartite graphs

CG232

14:00–14:20 **C. Merino**
On the number of tilings of rectangles with T-tetraminoes

14:25–14:45 **D. Stark**
Random preorders

14:50–15:10 **R.W. Whitty**
Rook polynomials on 2-dimensional surfaces

Summary of Thursday afternoon speakers

	CG93	CG60	CG83	CG85	CG232
14:00–14:20	Brandt	Montágh	Bullock	Abreu	Merino
14:25–14:45	Kaiser	Borg	Singleton	Labbate	Stark
14:50–15:10	Paulusma	Abbas	van Aardt	Martin	Whitty

Friday

07:45–08:30	Breakfast in Collingwood
09:15–10:15	Angelika Steger (Chair: Keith Edwards) CG93 *The sparse regularity lemma and its applications*
10:15–10:45	Refreshments
10:45–12:20	Contributed talks (4 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
12:30–13:30	Lunch in Collingwood
14.00–14.45	Contributed talks (2 slots) in 5 parallel sessions Rooms CG93, CG60, CG83, CG85, CG232
15:00–16:00	Ben Green (Chair: Peter Cameron) *Finite field models in additive combinatorics*
16:05–16:30	Refreshments
16:30	End of Conference

Friday morning contributed talks

CG93

10:45–11:05	**D.R. Woodall** Recent results on total choosability and edge colourings	
11:10–11:30	**C. Greenhill** Bounds on the generalised acyclic chromatic numbers of bounded degree graphs	
11:35–11:55	**H. Bielak** Chromatic zeros for some medial graphs	
12:00–12:20	**T.J. Rackham** Local nature of Brooks' colouring	

CG60

10:45–11:05	**V.I. Levenshtein** Reconstruction of graphs from metric balls of their vertices	
11:10–11:30	**N. López** Eccentricity sequences and eccentricity sets in digraphs	
11:35–11:55	**P. van den Berg** The number of edges in a bipartite graph of given order and radius	
12:00–12:20	**M. Aïder** Balanced almost distance-hereditary graphs	

CG83

10:45–11:05	**C. McDiarmid** Random planar graphs and related structures	
11:10–11:30	**B.D. McKay** Short cycles in random regular graphs	
11:35–11:55	**D.B. Penman** Extremal Ramsey graphs	
12:00–12:20	**A. Jamshed** A degree constraint for uniquely Hamiltonian graphs	

CG85

10:45–11:05 **L.K. Jørgensen**
Extremal results for rooted minor problems

11:10–11:30 **S. Bonvicini**
Live one-factorizations and mixed translations in even characteristic

11:35–11:55 **A. Bonisoli**
Factorizations with symmetry

12:00–12:20 **I.M. Wanless**
Perfect 1-factorisations and atomic Latin squares

CG232

10:45–11:05 **P. Danziger**
More balanced hill-climbing for triple systems

11:10–11:30 **M. Dewar**
Ordering the blocks of a design

11:35–11:55 **Y. Fujiwara**
Constructions for cyclic 4- and 5-sparse Steiner triple systems

12:00–12:20 **A.D. Forbes**
6-sparse Steiner triple systems

Summary of Friday morning speakers

	CG93	CG60	CG83	CG85	CG232
10:45–11:05	Woodall	Levenshtein	McDiarmid	Jørgensen	Danziger
11:10–11:30	Greenhill	López	McKay	Bonvicini	Dewar
11:35–11:55	Bielak	van den Berg	Penman	Bonisoli	Fujiwara
12:00–12:20	Rackham	Aïder	Jamshed	Wanless	Forbes

Friday afternoon contributed talks

CG93

14:00–14:20 **P.J. Cameron**
An orbital Tutte polynomial

14:25–14:45 **J.D. Rudd**
Orbits of graph automorphisms on proper vertex colourings

CG60

14:00–14:20 **P. Keevash**
The rôle of approximate structure in extremal combinatorics

14:25–14:45 **A. Sapozhenko**
On the number of independent sets in graphs

CG83

14:00–14:20 **H. Pollatsek**
Quantum error correction codes invariant under symmetries of the square

14:25–14:45 **S. Severini**
Permutations and Quantum Entanglement

CG85

14:00–14:20 **C.A. Baker**
Graphs with the n-e.c. adjacency property constructed from affine planes

14:25–14:45 **T.S. Griggs**
Steiner triple systems and existentially closed graphs

CG232

14:00–14:20 **T. Adachi**
Construction of a regular group divisible design

14:25–14:45 **H. Shen**
Mendelsohn 3-frames and embeddings of resolvable Mendelsohn triple systems

Summary of Friday afternoon speakers

	CG93	CG60	CG83	CG85	CG232
14:00–14:20	P. Cameron	Keevash	Pollatsek	Baker	Adachi
14:25–14:45	Rudd	Sapozhenko	Severini	Griggs	Shen

Abstracts of contributed talks

A family of large chordal ring of degree six

A. Abbas

(joint work with T. Bier)

MSC2000: 05C35

This paper discusses the covering property and the Uniqueness Property of Minima (UPM) for linear forms in an arbitrary number of variables, with emphasis on the case of three variables. It also studies the degree-diameter problem for undirected chordal ring graphs of degree six. We focus upon maximizing the number of vertices in the graph for given diameter and degree. We improve the result in [1] by finding that the family of triple loop graphs of the form $G(6d^2 - 2d + 1; 1; 3d + 1; 3d + 2)$ has a larger number of nodes for diameter d than the family $G(3d^2 + 3d + 1; 1; 3d + 1; 3d + 2)$ given in [1]. Moreover we show that both families have the Uniqueness Property of Minima. This paper is going to answer the following questions

(1) What is the maximum number of nodes in a chordal ring of degree six (triple loop graph) for given diameter d?

(2) What is Uniqueness Property of Minima (UPM)?

(3) What is the bound for number of nodes in the triple loop graph for diameter d ?

Reference.

[1] Yebra,J.L.A., Fiol, M.A., Morillo, P.,Alegre, I. The diameter of undirected graphs associated to plane tesselations. *Ars Combinatoria*, 20B(1985), pp. 159-172.

Graphs and digraphs with all 2–factors isomorphic

M. Abreu

(joint work with R. Aldred, M. Funk, B. Jackson, D. Labbate, J. Sheehan)

MSC2000: 05C70, 05C75, 05C20

Let $\boldsymbol{U(k)}$ be the family of all (connected) k–regular graphs G such that G has a 2–factor and all 2–factors of G are isomorphic. We use $\boldsymbol{BU(k)}$ to denote the set of graphs in $U(k)$ which are also bipartite, $\boldsymbol{HU(k)}$ is the set of graphs in $U(k)$ which are also hamiltonian, and $\boldsymbol{HBU(k)}$ are those graphs in $U(k)$ which are also hamiltonian and bipartite.

In previous works the coauthors proved that $BU(k) = \emptyset$ for $k \geq 4$ and constructed an infinite family of graphs in $HBU(3)$. Furthermore, they conjectured that all 3–connected graphs in $HBU(3)$ belong to this family and that all 3-connected graphs in $BU(3)$ belong to $HBU(3)$. Diwan has shown that there are no planar graphs in $HU(3)$.

Here we present the following results:

1. A digraph which contains a directed 2–factor and has minimum in–degree and out–degree at least four has two non-isomorphic directed 2–factors.

   And as a corollary

2. Every graph which contains a 2–factor and has minimum degree at least eight has two non-isomorphic 2–factors. This is $U(k) = \emptyset$ for $k \geq 8$.

In addition we construct: an infinite family of strongly connected 3–diregular digraphs with the property that all their directed 2–factors are isomorphic, an infinite family of 2–connected 4–regular graphs with the property that all their 2–factors are isomorphic, and an infinite family of cyclically 6–edge–connected cubic graphs with the property that all their 2–factors are hamiltonian cycles.

Construction of a regular group divisible design

T. Adachi

MSC2000: 05B05, 05B30, 05C20

In this talk, we characterize the combinatorial structure of some class of group divisible (GD) designs and develop some new construction methods leading to more GD designs.

For a singular GD design $r = \lambda_1$ holds, while in case of a nonsingular GD design $r > \lambda_1$ holds. It may be natural to investigate the case of $r = \lambda_1 + 1$, since it may have some interconnecting property (the next saturated case) between singular and nonsingular cases.

The combinatorial property of a GD design with $r = \lambda_1 + 1$ was first investigated by Shimata and Kageyama (2002) who showed that a GD design with $r = \lambda_1 + 1$ must be symmetric and regular. Jimbo and Kageyama (2003) completely characterized a GD design with $r = \lambda_1 + 1$ in terms of Hadamard tournaments and strongly regular graphs. Furthermore, Adachi, Jimbo and Kageyama (2003) characterized the combinatorial structure of GD designs without "α-resolution class" in each group in terms of Hadamard tournaments and strongly regular graphs. The result given by Jimbo and Kageyama (2003) is included in the result given by Adachi, Jimbo and Kageyama (2003) as a special case.

Here, we provide some constructions of regular GD designs based on such characterization.

On the Frobenius problem of three numbers: Part II

Francesc Aguiló

(joint work with A. Miralles and M. Zaragozá)

MSC2000: 05C20, 10A50, 11D04.

Given a set $A = \{a_1, ..., a_k\} \subset \mathbb{N}$, with $\gcd(a_1, ..., a_k) = 1$, let us define

$$R(A) = \{\sum_{i=1}^{k} \lambda_i a_i |\ \lambda_1, ..., \lambda_k \in \mathbb{N}\},$$

and $\overline{R}(A) = \mathbb{N} \setminus R(A)$. It can be easily seen that $|\overline{R}(A)| < \infty$. The *Frobenius problem* related to A, $\mathrm{FP}(A)$, consists on the study of the set $\overline{R}(A)$. The *solution* of $\mathrm{FP}(A)$ is the explicit description of $\overline{R}(A)$, however this is a difficult task. Usually *partial solutions* are given, like the cardinal $|\overline{R}(A)|$ and/or the *Frobenius number* $f(A) = \max \overline{R}(A)$.

We develop the ideas given in the Part I of this work to extend the method given there. In the first work a method to solve $\mathrm{FP}(A)$ is given, provided that the MDD related to A is a MDDE also. Now, in this second work, when the MDD is not a MDDE, we propose a technique to find the MDDE from the MDD.

Therefore parts I and II of this work give a generic method to solve the Frobenius problem of three numbers. To give some applications of the method, we solve several symbolical Frobenius problems which improve some known results in the bibliography.

Bipartite almost distance-hereditary graphs

Méziane Aïder

MSC2000: 05C12

The notion of distance-hereditary graphs has been extended to construct the class of almost distance-hereditary graphs (an increase of the distance by one unit is allowed by induced subgraphs). These graphs have been characterized in terms of forbidden induced subgraphs. Since the distance in bipartite graphs can not increase exactly by one unit, we have to adapt this notion to this case.

In this talk, we define the class of bipartite almost distance-hereditary graphs (an increase of the distance by two is allowed by induced subgraphs). We obtain a characterization of these graphs in terms of forbidden induced subgraphs.

Perfect graphs and vertex colouring problem of a graph

H. Ait Haddadene

MSC2000: 05C85, 68Q20

The concept of perfect graph was introduced by C.Berge at the beginning of the Sixties, introduced it while being interested in work of C.Schannon in information theory on the capacity of a channel of communication. He had defined:A graph γ-perfect (respectively α-perfect) as being a graph such as, for any induced subgraph H of G, the chromatic number γ(H) of H is equal to the size ω(H) of a largest clique of H (respectively cardinality minimum θ(H) of a cover by cliques of H is equal to the size α(H) of largest stable). It proposed two conjectures: The first known as being the weak conjecture of the perfect graphs was shown by L.Lovasz (1972) and was become since the theorem of perfect graph: "A graph G is γ-perfect if and only if G is α- perfect". These two concepts became since identical and are signed by perfect graph. The second is known as the strong perfect graph conjecture: A graph is perfect if and only if it contains no odd hole and no odd antihole (a hole is a chordless cycle of length at least four and an antihole is the complementary graph of a hole). M.Chudnovsky et al are proved the strong perfect graph conjecture in 2002 and it became the strong perfect graph theorem.A coloring of the vertices of a graph G with ω(G) colors is called optimal coloring or minimum coloring of the graph G. The problem to determine an optimal coloring of a graph is NP-complete. This problem becomes polynomial in the case of the perfect graphs. Grotschel et al (1984) developed polynomial algorithm to solve this problem for the whole of the perfect graphs. This algorithm uses an alternative of the method of the ellipsoids for the resolution of linear programs. The interest of the result of Grotschel et al is not algorithmic so much. Indeed their algorithm is not practically effective, because it do not take account of the combinatorial structure of perfect graphs. Thus, the search for very effective polynomial algorithms to solve this problem in the case of the perfect graphs or, more modestly, in subfamilies of perfect graphs continues to have a practical interest.In this paper, we will try to present the history of the advance of the study of perfect graphs and its bond with the vertex colouring problem of a graph. Our contribution in this framework and some bonds will be also presented.

Optical orthogonal oodes: new constructions

T.L. Alderson

(joint work with K. Mellinger)

MSC2000: 51E21, 51E14, 94A99

An $(n, w, \lambda_a, \lambda_c)$-optical orthogonal code (OOC) is a family of binary sequences (codewords) of length n, with constant Hamming weight w satisfying the following two conditions:

- (auto-correlation property) for any codeword $c = (c_0, c_1, \ldots, c_{n-1})$ and for any integer $1 \leq t \leq n-1$ there holds $\sum_{i=0}^{n-1} c_i c_{i+t} \leq \lambda_a$
- (cross-correlation property) for any two distinct codewords c, c' and for any integer $1 \leq t \leq n-1$ there holds $\sum_{i=0}^{n-1} c_i c'_{i+t} \leq \lambda_c$

where each subscript is reduced modulo n.
One of the first proposed applications of optical orthogonal codes was to optical code-division multiple access communication system where binary sequences with strong correlation properties are required. Subsequently, OOCs have found application for multimedia transmissions in fiber-optic LANs. Optical orthogonal codes have also been called cyclically permutable constant weight codes in the construction of protocol sequences for multiuser collision channels without feedback.
An $(n, w, \lambda_a, \lambda_c)$-OOC with $\lambda_a = \lambda_c$ is denoted (n, w, λ)-OOC. The number of codewords is the size of the code. For fixed values of n, w, and λ, the largest size of an (n, w, λ)-OOC is denoted $\Phi(n, w, \lambda)$. From the Johnson bound for constant weight codes it follows that

$$\Phi(n, w, \lambda) \leq \left\lfloor \frac{1}{w} \left\lfloor \frac{n-1}{w-1} \left\lfloor \frac{n-2}{w-2} \left\lfloor \cdots \left\lfloor \frac{n-\lambda}{w-\lambda} \right\rfloor \right\rfloor \cdots \right\rfloor \right.\right.\right. \tag{1}$$

(n, w, λ)-OOCs meeting this bound are said to be optimal. If C is an $(n, w, \lambda_a, \lambda_c)$-OOC with $\lambda_a \neq \lambda_c$ then we obtain a (perhaps naive) bound on the size of C by taking $\lambda = max\{\lambda_a, \lambda_c\}$ in (1).
For $\lambda = 1, 2$ optimal OOCs are known to exist. It is still unknown as to whether optimal codes exist with $\lambda > 2$. Certain families of conics in $PG(2, q)$ give rise to $(n, q+1, 2)$-OOCs which are close to optimal. We discuss generalizations whereby OOC's are constructed using Baer subplanes and families of arcs in PG(k,q). Among the codes constructed are some new large $(n, w, 3)$-OOC's.

Negative Hadamard Graphs

A. Alipour

MSC2000: 05B20, 11T71, 15A63, 94B25, 94B65

In 1985 Hadamard graphs were defined by Ito Noboru, [!], [2]. An Hadamard Graph $\Delta(n)$ is a graph whose vertices are all -1,1-vectors of length n and two vertices are adjacent if their inner product is zero. We note that there is an Hadamard matrix of order n if and only if the clique number of $\Delta(n)$ is n. In this paper we introduce the negative Hadamard graphs. Let $V_n = \{\pm 1\}^n$. We construct a graph Γ_n with vertex set V_n in which two vertices u and v are adjacent if $u \cdot v < 0$. We call this graph the negative Hadamard graph of order $n+1$. We prove that if the clique number of Γ_n is at least n, then it is $n+1$ and there is an Hadamard matrix of order $n+1$. Also we prove that this graph is vertex transitive and determine the domination number, the edge chromatic number and the structure of the automorphism group of this graph. In particular we prove that for $n \geq 4$ and $n \equiv 2$ or 3 (mod 4), the automorphism group of Γ_n is isomorphic to $S_n \times \mathbb{Z}_2^n$.

References.

[1] I. Noboru, Hadamard graphs. I. Graphs Combin. **1** (1985), no. 1, 57–64.

[2] I. Noboru, Hadamard graphs. II. Graphs Combin. **1** (1985), no. 4, 331–337.

A general approach to constructing power-sequence terraces for $\mathbb{Z}_n$

Ian Anderson

(joint work with D.A. Preece)

MSC2000: 11A07, 05B30

A terrace for $\mathbb{Z}_n$ is an arrangement $(a_1, a_2, \ldots, a_n)$ of the n elements of $\mathbb{Z}_n$ such that the sets of differences $a_{i+1} - a_i$ and $a_i - a_{i+1}$ $(i = 1, 2, \ldots, n-1)$ between them contain each element of $\mathbb{Z}_n \setminus \{0\}$ exactly twice. For n odd, many procedures have been published for constructing power-sequence terraces for $\mathbb{Z}_n$; each such terrace may be partitioned into segments one of which contains merely the zero element of $\mathbb{Z}_n$ whereas each other segment is either (a) a sequence of successive powers of an element of $\mathbb{Z}_n$ or (b) such a sequence multiplied throughout by a constant. We now present a new general power-sequence approach that yields $\mathbb{Z}_n$ terraces for all odd primes n less than 1000 except for $n = 601$. It also yields terraces for some groups $\mathbb{Z}_n$ with $n = p^2$ where p is an odd prime, and for some $\mathbb{Z}_n$ with $n = pq$ where p and q are distinct primes greater than 3. Each new terrace has at least one segment consisting of successive powers of 2, modulo n.

On the structure of equireplicate partial linear spaces with constant line size

John Arhin

MSC2000: 05B15

A *partial linear space* $\mathcal{S} = (\mathcal{P}, \mathcal{L})$ consists of a set $\mathcal{P}$ of *points* together with a set $\mathcal{L}$ of *lines*, where each line is a subset of $\mathcal{P}$ (of cardinality greater than or equal to 2), such that every pair of points is contained in at most one line.

A partial linear space is said to be *equireplicate* if every point is contained within the same constant number of lines. We then call this constant the *replication number*.

A *PLS*$(v, n; r)$ is a equireplicate partial linear space, where the set of points has size v, each line has size n and the replication number is r.

Let $\mathcal{S} = (\mathcal{P}, \mathcal{L})$ be a PLS$(v, n; r)$.

A *decomposition* of $\mathcal{S}$ is a partition $\{\mathcal{L}_1, \dots, \mathcal{L}_m\}$ of $\mathcal{L}$, such that each $(\mathcal{P}, \mathcal{L}_i)$ is an equireplicate partial linear space.

Note that by the definition of $\mathcal{S}$, each $(\mathcal{P}, \mathcal{L}_i)$ is a PLS$(v, n; r_i)$, for some r_i.

Now $\{\mathcal{L}\}$ is one decomposition of $\mathcal{S}$. If $\{\mathcal{L}\}$ is the only decomposition of $\mathcal{S}$, then $\mathcal{S}$ is said to be *indecomposable*; otherwise $\mathcal{S}$ is said to be *decomposable.*

An *unrefinable decomposition* of $\mathcal{S}$ is a decomposition $\{\mathcal{L}_1, \dots, \mathcal{L}_m\}$ of $\mathcal{S}$, such that each $(\mathcal{P}, \mathcal{L}_i)$ is indecomposable.

In this talk, we discuss the result that every PLS$(n^2, n; r)$ has a unique unrefinable decomposition, and provide an efficient algorithm to compute it. Not only does this result imply that every affine plane has a unique unrefinable decomposition, but it also has important implications for the structure of SOMAs (a generalisation of mutually orthogonal Latin squares). We then briefly look at the structure of a PLS$(v, n; r)$, when $v < n^2$ and $v > n^2$.

Permutation groups, error-correcting codes and uncoverings

R.F. Bailey

MSC2000: 94B99, 20B20, 05B40

We replace the traditional setting of error-correcting codes (namely vector spaces over finite fields) with that of permutation groups, using permutations written in list form as the codewords. We will describe some groups which are suitable for this purpose, and introduce a decoding algorithm which in turn uses what we call an *uncovering*. These are objects which are closely related to covering designs.

This is a continuation of work presented at BCC19 in 2003.

Graphs with the n-e.c. adjacency property constructed from affine planes

C.A. Baker

(joint work with A. Bonato, J.M.N. Brown, and T. Szőnyi)

MSC2000: 05C99, 05B25, 05C80

Adjacency properties of graphs were first studied by Erdős and Rényi in their classic work on random graphs. One such adjacency property is the n-existentially closed property: for a positive integer n, a graph G is *n-existentially closed* or *n-e.c.* if for all n-subsets S of vertices of G, and all subsets T of S, there is a vertex not in S joined to all the vertices of T and not joined to any of the vertices in $S \setminus T$. Erdős and Rényi proved in 1963 that almost all graphs (with fixed edge probability $0 < p < 1$) are n-e.c. Despite this fact, few explicit classes of graphs with the n-e.c. property are known. In 1981, Bollobás and Thomason proved that sufficiently large Paley graphs are n-e.c., while P. Cameron and Stark recently used affine designs and probabilistic methods to construct examples of many non-isomorphic strongly regular n-e.c. graphs.

We use methods from finite geometry to construct new examples of n-e.c. graphs. Our techniques employ collinearity graphs of partial planes derived from even order affine planes. The strongly regular graphs we obtain are distinct from both the Paley graphs and the graphs of Cameron and Stark. In addition, our proofs (unlike proofs for earlier constructions) are elementary in that they do not use any specialized machinery beyond basic properties of affine planes, counting, and probability theory. If time permits, then we will describe a new n-e.c. preserving operation using switching. For certain orders the new operation provides an exponential number of non-isomorphic n-e.c. graphs.

Consecutive magic graphs

C. Balbuena

(joint work with K.C. Das, Y. Lin, M. Miller, J. Ryan, Slamin, K. Sugeng, M. Tkac)

MSC2000: 05C78

Let G be a graph with order n and size e. A vertex-magic total labelling is an assignment of the integers $1, 2, \ldots, n+e$ to the vertices and the edges of G so that at each vertex, the vertex label and the labels on the edges incident at that vertex add to a fixed constant called magic number of G. Such a labelling is a-vertex consecutive magic if the set of the labels of the vertices is $\{a+1, a+2, \ldots, a+n\}$, and is b-edge consecutive magic if the set label of edges is $\{b+1, b+2, \ldots, b+e\}$. In this paper we prove that every a-vertex consecutive magic graph, other than the union of a vertex and a path of length two, has degree at least one and at least as many edges as the number of vertices minus one. As a consequence, we show that every tree with even order is not a-vertex consecutive, and if a tree of odd order is a-vertex consecutive, then $a = n - 1 = e$. Furthermore, we show that every a-vertex magic graph with $e > n$ and n odd, or $2e \notin \{3n-2, 3n\}$ and n even, has minimum degree two if $a < e$, or has minimum degree three if $a < (e-n-1)/2$. Finally, we state analogous results for b-edge consecutive magic graphs.

A new approach to finite semifields

Simeon Ball

(joint work with Michel Lavrauw and Gary Ebert)

MSC2000: 51E15

A *finite pre-semifield* is a finite set S with two operations, addition $(+)$ and multiplication $(\circ)$, such that $(S,+)$ is an additive group, both distributive laws hold and

$$x \circ y = 0 \text{ implies } x = 0 \text{ or } y = 0.$$

A pre-semifield can be used to coordinatise a projective plane of order $|S|$ and we are interested in finding pre-semifields that produce non-isomorphic projective planes. Two pre-semifields are said to be *isotopic* if they coordinatise isomorphic planes. A *semifield* is a pre-semifield that has a multiplicative identity. There is always a semifield isotopic to any pre-semifield. There are less than roughly 20 known families of (mutually non-isotopic) semifields. Unless it is immediate that two semifields are not isotopic it is generally difficult to establish whether or not they are. Above all, the goal in this area is to construct many more families of non-isotopic semifields. The first semifields were discovered by Dickson, roughly 100 years ago with more examples given later by Albert (1950's), Knuth (1960's), Cohen and Ganley (1980's) and various families due to Kantor, amongst others, have been constructed in the last twenty years.

It can be shown that $|S| = q^n$ for some prime power q and that S can viewed as a vector space of rank n over $\mathbb{F}_q$, where multiplication is given by $a_{ijk} \in \mathbb{F}_q$ by the rule

$$e_i \circ e_j = \sum_{k=1}^{n} a_{ijk} e_k,$$

where $\{e_1, e_2, \ldots, e_n\}$ is a basis for S over $\mathbb{F}$. Knuth was first to note that any permutation of the subscripts produces another semifield, so there are six semifields associated with any semifield.

In this talk I shall present a new way to construct finite semifields of order q^n from two subspaces of a vector space of rank $(r+1)n$ over $\mathbb{F}_q$, for some r. In fact any finite semifield can be constructed in this way for some $r \le n-1$, moreover [probably] all known semifield of order q^n can be constructed from two subspaces of a vector space of rank $2n$ or rank $3n$ over $\mathbb{F}_q$.

The construction also provides us with a new operation (not one of the six due to Knuth) which produces more semifields in the case when $r = 1$.

Group Key Distribution Patterns

J.C. Bate

(joint work with SeonHo Shin)

MSC2000: 05B30, 51E30

Key Distribution Patterns (KDPs), as introduced by Mitchell and Piper in 1987 provide an efficient method of secure communication between every pair of users in a large network. Every user in the network stores a small set of subkeys and the key required for a pair of users to communicate securely can be made up from a combination of some of the subkeys already held in common by that pair.

However, what if it wasn't every pair of users wishing to communicate, but some other predefined subsets of users from within the network?

Group Key Distribution Patterns are generalized KDPs displaying many interesting characteristics inherited from Mitchell and Pipers KDPs, whilst at the same time providing a method of secure communication between all predefined subsets of users from within the network.

On graphs with least eigenvalue -2

F.K. Bell

(joint work with E.M. Li Marzi and S.K. Simić)

MSC2000: 05C50

Let μ be an eigenvalue of a graph G, with multiplicity k. A *star complement* for μ in G is an induced subgraph $H = G - X$, where $|X| = k$ and μ is not an eigenvalue of H.

The class of graphs with least eigenvalue -2 has been studied extensively in recent years. We characterise the possible star complements for -2 of such graphs.

Validation of a particular class of bilinear systems

F. Benmakrouha

(joint work with C. Hespel)

MSC2000: 05A05

We study the validation of a family (B_k) of bilinear system, global modelling of an unknown dynamical system (Σ).

Two formal power series in noncommutative variables are used for describing (Σ) : the generating series for the system's behavior (G) and the Chen series for the system's input. The family (B_k) of bilinear systems is described by its rational generatrice series (G_k) such that the coefficients of (G) and (G_k) coincide up to order k.

We propose a symbolic computation of coefficients of (G_k). These coefficients are powers of an operator Θ which is in the monoid generated by two linear differential operators Δ and Γ .

We give according to [1] a combinatorial interpretation of these powers. The n-th power of Θ is equal to the sum of the labels of all forests of increasing trees on $\{1, \cdots, n\}$.

Bounding these coefficients, one obtains an estimation of the error due to the approximations by (B_k). This error computation allows one to better measure the impact of noisy inputs on the convergence of (B_k). Indeed, one can determine the contribution of the inputs and of the system in the error computation.

Reference.

[1] F. Bergeron,C Reutenauer,*Combinatorial interpretation of the powers of a linear differentiel operator* Rapport de recherche Université du Québec Montréal. Mars 1986.

Cycle regularity and Hypercubes

A. Berrachedi

(joint work with N. Kahoul)

MSC2000: 05C15, 05C17, 05C69, 05C85

It is known that the subgraph H_k, induced by the *two* central levels of the hypercube of odd degree $2k-1$, is of maximum order among the graphs, in which each path of length three belongs to one single cycle of length *three*. In this paper, we show that H_k is of maximum diameter in the same class. Moreover, we consider graphs for which each induced path of length three, with distinct ends belongs to exactly one induced cycle of length three. This class generalizes the class defined above and contains the hypercubes. We give several properties for these classes of graphs and a new characterization of H_K.

References.

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[3] M.Mollard: Quelques problèmes combinatoires sur l'Hypercube et les graphes de Hamming, Thèse Doctorat es-Science, Université Joseph Fourier, Grenoble 1989.

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Chromatic zeros for some medial graphs

H. Bielak

MSC2000: 05C15

The chromatic polynomial $P(G, \lambda)$ of a graph G in the variable λ counts for positive integers λ the proper vertex $\lambda-$colourings of G.

In this paper we study the location of chromatic complex zeros (i.e., zeros of chromatic polynomials) for some medial graphs. A medial graph $M(G)$ is defined for a plane embedding of a planar graph $G = (V(G), E(G))$ as follows: $V(M(G)) = E(G)$ and two vertices in $M(G)$ are adjacent if and only if the respective edges are incident and belong to the boundary of the same region of G. In particular, we give an infinite family of hamiltonian medial graphs with chromatic complex zeros in the disk $|z - 3/2| \leq r$, where $r > r_0$ and $r_0 - 1/2$ is maximum positive root of the equation $x^4 - 2x^3 - 5x^2 - 6x - 1 = 0$. Note that r_0 belongs to the interval $(4.25, 4.375)$.

Equipartite and almost-equipartite gregarious 4-cycle systems

Elizabeth J. Billington

(joint work with Dean Hoffman)

MSC2000: 05B30, 05C38

Let $K_{n(m)}$ denote a complete multipartite graph with n parts of size m, and let $K_{n(m),t}$ denote a complete multipartite graph with $n+1$ parts: n of size m and one of size t.

A gregarious 4-cycle decomposition of a complete multipartite graph is a decomposition into 4-cycles such that each cycle has its four vertices in different partite sets of the graph (as long as the complete multipartite graph has at least four parts). If there are precisely four parts, it is easy to see that they must all have the same (even) size. However, this is not so if there are more than four parts. Here we consider gregarious 4-cycle decompositions of the graphs $K_{n(m)}$ and $K_{n(m),t}$, and show existence of such a decomposition if and only if there is an "ordinary" 4-cycle decomposition, and in the case of the graph $K_{n(m),t}$, the part of size t is bounded: $t \leq m(n-1)/2$.

Factorizations with symmetry

A. Bonisoli

MSC2000: 05C70, 51E20

Let $\mathcal{F}$ be a 1–factorization of K_{2n}. Already in the seventies it was pointed out that a random choice of $\mathcal{F}$ will probably yield an object with a total lack of symmetry, which means $Aut(\mathcal{F}) = \{\text{id}\}$.

On the other hand, the "standard" textbook constructions aiming to show that a 1–factorization of K_{2n} exists for an arbitrary value of n are usually based on symmetry arguments: typically a single 1–factor is constructed, the other ones are obtained from it by rotation or reflection. Similar arguments hold when $\mathcal{F}$ is a 2–factorization of K_v, v odd.

An automorphism group G of $\mathcal{F}$ is by definition a permutation group on the set of vertices of the complete graph leaving the factorization invariant, regardless of whether $\mathcal{F}$ is a 1–factorization or a 2–factorization or even some other kind of decomposition. Consequently G acts on the vertices but also on the edges of the complete graph and on the factors of the factorization.

It was precisely by imposing conditions on these actions that the best classification results were obtained, as in the case of 1–factorizations admitting an automorphism group acting doubly transitively on vertices. On the other hand, even the most powerful construction techniques such as those based on starters do require some group acting in a prescribed manner.

Does there exist a primitive 1–factorization of K_{2n} which is not doubly transitive? Is it possible to have a construction for infinitely many values of n? These questions have affirmative answers.

Doubly transitive 2–factorizations of K_v have been recently classified. It can be shown that they all arise from the affine line–parallelism of $AG(d, p)$ for some odd prime p in a standard manner. The assumption can be weakened to "doubly homogeneous."

Live one–factorizations and mixed translations in even characteristic

S. Bonvicini

MSC2000: 05C70, 51E20

A one–factorization $\mathcal{F}$ of K_{2n} is said to be *live* if each one of its one–factors, when viewed as a fixed–point–free involution on the set of vertices, induces an automorphism of $\mathcal{F}$.

The affine line–parallelism of $AG(d, 2)$ is an example of such a one–factorization, since the involution corresponding to a class of parallel lines yields a translation.

In this talk we present an example of a live one–factorization which is NOT an affine line–parallelism. To this purpose we develop the notion of a *mixed* translation in $AG(d, 2)$, that is namely a transformation which suitably moves the points of a given hyperplane in one direction and the points of the complementary hyperplane in another direction. These transformations always come in pairs. If we replace the two translations in the given directions by the corresponding pair of mixed translations we obtain the required live one–factorization. In geometric terms, that amounts to replacing half of the lines in one parallel class by the lines of the other parallel class in the complementary hyperplane.

Do live one–factorizations exist when the number of vertices is NOT a power of 2?

Graphs with the Erdős-Ko-Rado property

Peter Borg

(joint work with Fred Holroyd)

MSC2000: 05C35, 05D05

A pairwise intersecting family of sets is *non-centred* if the intersection of all sets of the family is empty. A graph G is said to have the *(strict) Erdős-Ko-Rado property*, or to be *(strictly) r-EKR*, if no such family of independent r-sets of vertices is (as large as) larger than the largest family of independent r-sets containing v for any vertex v.

The Erdős-Ko-Rado Theorem states that E_n is r-EKR for all $r \leq n/2$, and strictly so for $r < n/2$, where E_n is the empty graph on n vertices. Holroyd and Talbot conjectured that if ($r < \mu(G)/2$) $r \leq \mu(G)/2$ then G is (strictly) r-EKR, where $\mu(G)$ denotes the minimum cardinality of a maximal independent set of G. Apart from empty graphs, this conjecture is known to hold for graphs that belong to some other classes. For example, Holroyd and Talbot demonstrated the conjecture for the case when G is a disjoint union of two complete multipartite graphs, and they also showed that G is strictly r-EKR for $r < \mu(G)/2$.

We verify the conjecture for the case when G is a disjoint union of an arbitrary number of complete multipartite graphs and at least one isolated vertex. We also distinguish all the cases in which such a graph is strictly r-EKR or not, when $r = \mu(G)/2$.

Bell's number in the Alekseev inequality

S. Bouroubi

MSC2000: 06B30

Let P_n be the partition lattice on n elements and let r be it's rank function. A representation of P_n is a function $X : P \to \mathbb{R}$ so that $p > q$ implies $X(p) - X(q) \geq 1$. The mean and the variance of X are defined respectively by :

$$\mu_X = \frac{1}{B_n} \sum_{\pi \in P_n} X(\pi) \;\; \& \;\; \sigma_X^2 = \frac{1}{B_n} \sum_{\pi \in P_n} (X(\pi) - \mu_X)^2$$

where B_n denotes the n^{th} Bell number.

A representation X^* is said to be optimal if $\sigma_{X^*}^2 \leq \sigma_X^2$ for every representation X of P.

V.B. Alekseev showed that r is optimal iff

$$\mu_r(F) \geq \mu_r, \text{ for every filter } F \text{ of } P_n \tag{1}$$

In this work we present a proof of the Alekseev inequality (1) on every filter generated by one element, using some new properties of the Bell's number sequence.

Triangle-free graphs whose independence number equals the degree

Stephan Brandt

MSC2000: 05C15, 05C35

In a triangle-free graph, the neighbourhood of every vertex is an independent set. We will investigate the class $\mathcal{S}$ of triangle-free graphs where the neighbourhoods of vertices are maximum independent sets. Such a graph G must be regular of degree $d = \alpha(G)$ and the fractional chromatic number must satisfy $\chi_f(G) = |G|/\alpha(G)$. We indicate that $\mathcal{S}$ is a rich family of graphs by showing that for every rational number c between 3 and 4 and for every rational number $c \geq 30/7$ there is a graph $G \in \mathcal{S}$ with $\chi_f(G) = c$. For $4 < c < 30/7$ we can only prove that the conclusion is true for a dense subset of the rational numbers in this range. For $2 \leq c < 3$, only constants of the type $c = (3i - 1)/i$ can be fractional chromatic numbers of graphs in $\mathcal{S}$ for positive integers i.

The statements for $c \geq 3$ are obtained by using, modifying, and reanalysing constructions of Sidorenko, Mycielski, and Bauer, van den Heuvel and Schmeichel, while the case $c < 3$ is settled by a recent result of Brandt and Thomassé. We will also investigate the relation of other parameters of certain graphs in $\mathcal{S}$ like chromatic number and toughness.

Matchings, Tutte sets, and independent sets

H.J. Broersma

(joint work with D. Bauer, A. Morgana, and E. Schmeichel)

MSC2000: 05C70, 05C75

We define a Tutte set of a graph $G = (V, E)$ as a set $S \subseteq V$ such that $\omega_0(G-S)-|S| = def(G) = \max_{X \subseteq V}\{\omega_0(G-X)-|X|\}$, where the maximum is taken over all proper subsets of V, where $\omega_0(G)$ denotes the number of odd components, and where $def(G)$ denotes the deficiency of G. By classical results due to Tutte and Berge, $def(G)$ is equal to the number of vertices of G unmatched by a maximum matching in G. We study maximal Tutte sets, and introduce the D-graph $D(G)$ of a graph with a perfect matching. We use the Edmonds-Gallai decomposition of a graph G to show how maximal Tutte sets in G relate to maximal independent sets in $D(G)$, and we characterize isomorphisms between iterated D-graphs. As a surprising consequence we obtain that $D^3(G) \cong D^2(G)$ for every graph G with a perfect matching.

Connected, nontraceable detour graphs

F.E.S. Bullock

(joint work with M. Frick and G. Semanišin)

MSC2000: 05C38

A graph G such that each vertex of G is an endvertex of a longest path in G is called a *detour* graph. The difference between the order of G and the order of a longest path in G is called the *detour deficiency* of G, and a detour graph with detour deficiency zero is called *homogeneously traceable*. Detour graphs are therefore a natural generalisation of homogeneously traceable graphs. Nonhamiltonian, homogeneously traceable graphs were investigated by Skupień in [2] and by Chartrand, Gould and Kapoor in [1].

In this talk we consider connected detour graphs with detour deficiency greater than zero. There are no such graphs with order less than 10, but we give constructions for connected detour graphs of all orders greater than 17 and all detour deficiencies greater than zero.

References.

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Colouring even cycle systems

Andrea C. Burgess

(joint work with David A. Pike)

MSC2000: 05C15, 05B30

An m-cycle system of order n is a partition of the edges of the complete graph K_n into m-cycles. An m-cycle system S is said to be weakly k-colourable if its vertices may be partitioned into k sets (called colour classes) such that no m-cycle in S has all of its vertices the same colour. The smallest value of k for which a cycle system S admits a weak k-colouring is called the chromatic number of S. We study weak colourings of even cycle systems (i.e. m-cycle systems for which m is even), and show that for any integers $r \geq 2$ and $k \geq 2$, there is a $(2r)$-cycle system with chromatic number k.

Max-algebra: the linear algebra of combinatorics?

Peter Butkovič

MSC2000: 15A15, 90C27

Let $a \oplus b = \max(a, b)$ and $a \otimes b = a + b$ for $a, b \in \overline{\mathbb{R}} := \mathbb{R} \cup \{-\infty\}$. By max-algebra we understand the analogue of linear algebra developed for the pair of operations $(\oplus, \otimes)$ extended to matrices and vectors formally in the same way as in linear algebra, that is if $A = (a_{ij})$, $B = (b_{ij})$ and $C = (c_{ij})$ are matrices with elements from $\overline{\mathbb{R}}$ of compatible sizes, we write $C = A \oplus B$ if $c_{ij} = a_{ij} \oplus b_{ij}$ for all i, j, $C = A \otimes B$ if $c_{ij} = \sum_k^{\oplus} a_{ik} \otimes b_{kj}$ for all i, j and $\alpha \otimes A = (\alpha \otimes a_{ij})$ for $\alpha \in \overline{\mathbb{R}}$.

We present an overview of strong links between max-algebraic problems and combinatorial or combinatorial optimisation problems. These links indicate that max-algebra may be regarded as a linear-algebraic encoding of a class of combinatorial problems. Instances of such problems are: the set covering (which in max-algebra is the solvability of a linear system), the minimal set covering (unique solvability of a linear system), existence of a directed cycle (strong regularity of a matrix), existence of an even directed cycle (regularity of a matrix), maximal cycle mean (eigenvalue), longest-distances (eigenvectors), best principal submatrices (coefficients of a characteristic polynomial), transitive closure (matrix power series), etc. Due to these links, max-algebra enables in some cases to find connections between combinatorial problems which would otherwise not be visible. A selection of open problems will be provided.

Coflow and covering vertices by directed circuits

Kathie Cameron

(joint work with Jack Edmonds)

MSC2000: 05C38, 05C70, 90C27, 68R10

Let G be any digraph such that each edge and each vertex is in a dicircuit. Let $d(v)$ be non-negative integers for vertices v, and $d(e)$ be non-negative integers for edges e. The capacity $d(C)$ of a dicircuit C means the sum of the d's of the vertices and edges in C. A version of the Coflow Theorem (1982) says:

The max cardinality of a subset S of the vertices of G such that each dicircuit C of G contains at most $d(C)$ members of S

equals

the minimum of the sum of the capacities of any subset H of dicircuits of G plus the number of vertices of G which are not in a dicircuit of H.

A feedback set in G means a subset F of its edges (minimal by inclusion) such that $G - F$ is acyclic. It is interesting to apply the Coflow Theorem to G and a feedback set F by letting $d(e) = 1$ for each e in F and letting the other d's be 0.

A feedback set F is called coherent if every edge of G is in some dicircuit which contains at most one member of F. That any G has a coherent feedback set is equivalent to a theorem of Bessy and Thomassé. Applying the Coflow Theorem to G with a coherent F yields immediately the following recent theorem of Bessy and Thomassé, conjectured by Gallai in 1963:

For any digraph G such that each edge and each vertex is in a dicircuit, the maximum number of vertices in G such that no two of them are joined by an edge is at least as big as the minimum number of dicircuits which together cover all the vertices.

An orbital Tutte polynomial

P.J. Cameron

(joint work with B. Jackson and J. Rudd)

MSC2000: 05C15, 05C25, 20B25

The two-variable Tutte polynomial of a graph Γ specialises to one-variable polynomials which count the numbers of nowhere-zero flows or tensions on Γ with values in an abelian group A. (These numbers depend only on the order of A, not on its structure.) We present a polynomial in two infinite sets of variables which specialises in the same way to polynomials counting the number of orbits of G on nowhere-zero tensions or flows, where G is a group of automorphisms of Γ. In this more general case, the numbers depend on the structure of A; specifically, we substitute for the ith variable in the polynomial the number of solutions of $ia = 0$ in the group A. Some properties of these polynomials, and some generalisations, will also be mentioned.

A superlinear lower bound for the size of a critical set in a latin square

N.J. Cavenagh

MSC2000: 05B15

A critical set is a partial latin square that has a unique completion to a latin square, and is minimal with respect to this property. In this talk we outline a proof that any critical set in a latin square of order n has size at least $n(\log n)^{1/3}/2$.

Average degree and extremal problems for infinite graphs

M. Cera

(joint work with C. Balbuena, A. Diánez and A. Márquez)

MSC2000: 05C35, 05C99

There exist many problems in Extremal Graph Theory for finite graphs relating the number of vertices to the number of edges, and therefore, related to the average degree. In this paper, we extend the concept of average degree for a family of infinite graphs that we call *average-measurable*. For infinite graphs we also extend the problem of determining the maximum number of edges of such a graph with no subgraph homeomorphic to a complete graph. Besides we study the relationship between this problem and the same problem in finite graphs.

References.

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Connectedness of graphs of 3-colourings

Luis Cereceda

(joint work with Jan van den Heuvel and Matthew Johnson)

MSC2000: 05C15, 05C85, 05C40

For a vertex 3-colourable graph G, let $\mathcal{C}_3(G)$ be the graph of 3-colourings of G. This is the graph with node set the proper 3-colourings of G, and two nodes adjacent whenever the corresponding colourings differ on precisely one vertex of G. Given G, what can we say about the structure of $\mathcal{C}_3(G)$? In particular, how easily can we decide if $\mathcal{C}_3(G)$ is connected? We give necessary and sufficient conditions for $\mathcal{C}_3(G)$ to be connected in terms of the structure and possible 3-colourings of G, and consider the complexity of this decision problem for various classes of 3-colourable graphs.

Admissible permutations for constructing Trojan squares for $2n$ treatments with odd-prime n side

P.E. Chigbu

MSC2000: 05B15, 62K05

The $(n \times n)/2$ Trojan squares for odd-prime n side are examined with the view of establishing the admissible permutations of the symmetric group, S_n, for directly constructing them via the group-theoretic approach of Bailey and Chigbu (1997). The unique group properties of the admissible permutations are also made evident while an algorithm, which would determine these permutations for constructing the Trojan squares is given and automated.

The ultimate isometric path number of a graph

Nancy E. Clarke

MSC2000: 05C70, 05C99, 91A43

The game of Cops and Robber is a pursuit game played on a reflexive graph. The cops choose vertices to occupy, then the robber chooses a vertex. The two sides then move alternately, where a move is to slide along an edge or along a loop, i.e. pass. Both sides have perfect information, and the cops win if any of the cops and the robber occupy the same vertex at the same time. The minimum number of cops that suffice to win on a graph G is the *copnumber* of G. The game has been considered on infinite graphs but, in this talk, we only consider finite graphs.

We consider the Cops and Robber game when the cops are restricted to moving on assigned "beats" or subgraphs, and bound the copnumbers of powers of graphs under a variety of products. In many cases, the results are shown to be asymptotically exact.

Covering Arrays of Strength Two

Charles J. Colbourn

MSC2000: 05B15

A covering array $CA(N; t, k, v)$ is an $N \times k$ array whose entries are from a v-set, in which every $N \times t$ subarray contains (as a row) every ordered t-tuple of the v symbols at least once. Recent research on covering arrays of strength two has focussed on

- improved product-type constructions (Colbourn, Martirosyan, Mullen, Shasha, Sherwood, and Yucas (2005));
- effective heuristic search techniques (Cohen (2004); Nurmela (2004));
- using automorphisms to accelerate computational search (Meagher and Stevens (2004); Meagher (2004); Colbourn (2005)); and
- constructions from orthogonal arrays.

This flurry of activity has had the unfortunate effect of making it quite difficult to determine the utility of the various constructions, since existing tables are out-of-date and restricted to very small orders. In this talk we therefore describe the computation of tables for the smallest N for which a $CA(N; 2, k, v)$ exists whenever $3 \leq k \leq 20000$ and $3 \leq v \leq 25$. In the process, we describe a new method for producing covering arrays of non-prime-power order v from orthogonal arrays of larger prime-power order.

Bounds on element order in rings Z_m with divisors of zero

Charlie H. Cooke

MSC2000: 13M99, 11A99

If p is a prime, integer ring Z_p has exactly $\phi(\phi(p))$ generating elements ω, each of which has maximal index $l_p(\omega) = \phi(p) = p - 1$. But, if $m = \Pi_{J=1}^{R} p_j^{a_j}$ is composite, it is possible that Z_m does not possess a generating element; and the maximal index of an element is not easily discernible. Here it is determined when, in the absence of a generating element, one can still with confidence place bounds on the maximal index. Moreover, general information about existence or non-existence of a generating element often can be predicted from the bound. A result is established which greatly reduces in the computational requirements for numerically deciding whether ring Z_m has a generating element.

Ovoids of the Hermitian surface and derivations

A. Cossidente

(joint work with G. Marino)

MSC2000: 51E21, 51E14

Some new derivation techniques of ovoids of the Hermitian surface $\mathcal{H}(3, q^2)$ of $\mathrm{PG}(3, q^2)$ are introduced and discussed. Moreover, connections between a special class of ovoids of $\mathcal{H}(3, q^2)$ and spreads of $\mathrm{PG}(3, q)$ are presented.

Signless Laplacians and line graphs

D. Cvetković

MSC2000: 05C50

The spectrum of a graph is the spectrum of its adjacency matrix. Cospectral graphs are graphs having the same spectrum. In this paper we study the phenomenon of cospectrality in graphs by comparing characterizing properties of spectra of graphs and spectra of their line graphs. We present some arguments showing that the latter perform better. In this comparison we use spectra of signless Laplacian (the adjacency matrix modified by putting vertex degrees on the diagonal) of graphs. Some properties of eigenvalues of signless Laplacian are given.

Distance and Inverse Degree

P. Dankelmann

(joint work with H.C. Swart and P. van den Berg)

MSC2000: 05C12

Let $G = (V, E)$ be a connected, finite graph of order n. The *average distance* $\mu(G)$ of G is defined as the average of the distances between all unordered pairs of vertices, The *inverse degree* $R(G)$ of G, is defined as the sum of the inverses of the degrees of the vertices of G,

$$R(G) = \sum_{u \in V} \frac{1}{(\deg v)},$$

where $\deg v$ is the degree of v in G.
The computer program GRAFFITI conjectured that $\mu(G) \leq R(G)$. Erdös, Pach, and Spencer proved the upper bound on the diameter of G,

$$\operatorname{diam}(G) \leq (6R(G) + o(1)) \frac{\log n}{\log \log n},$$

which, by $\mu(G) \leq \operatorname{diam}(G)$, is also an upper bound on the average distance. Moreover, they constructed an infinite family of graphs with average distance at least $\frac{2R(G)\log n}{9 \log \log n}$, thus disproving the GRAFFITI conjecture.
In our talk, we improve the upper bound by a factor of approximately 2.

More balanced hill-climbing for triple systems

P. Danziger

(joint work with D. Heap and E. Mendelsohn)

MSC2000: 05B07

Exhaustive enumeration of Steiner triple systems is not feasible, due to the the combinatorial explosion of instances. The next-best hope is to quickly find a sample that is representative of isomorphism classes. Stinson's hill-climbing algorithm certainly finds a sample quickly, but we find that the sample is far from uniformly distributed with respect to the isomorphism classes of the STSs, at least for $v \leq 19$. No analysis of the non-uniformity of the distribution with respect to isomorphism classes or the intractability of obtaining a representative sample for $v > 19$ is known.

We also investigate some modifications to hill-climbing that make the sample it finds closer to the uniform distribution over isomorphism classes without unduly degrading its performance.

The lattice of cyclic flats of a matroid

Anna de Mier

(joint work with J. Bonin)

MSC2000: 05B35

The lattice of flats of a matroid is a well-understood object: it is a geometric lattice, and every geometric lattice is the lattice of flats of a matroid. In this talk we focus on a particular type of flats, cyclic flats, which also give rise to a lattice. A flat of a matroid is called *cyclic* if it is a union of circuits. It is easy to check that cyclic flats form a lattice under inclusion. But this lattice is far from having the nice properties that the lattice of flats has; for instance, it is not necessarily geometric and all maximal chains need not have the same length. We show that in fact every lattice is isomorphic to the lattice of cyclic flats of some matroid (and moreover, of a matroid that is both transversal and cotransversal).

A matroid is uniquely determined by the set of its cyclic flats together with their ranks. We give a necessary and sufficient condition for a family of sets $\mathcal{Z}$ and a function $\rho : \mathcal{Z} \to \mathbf{N}$ to be the collection of cyclic flats of a matroid and their ranks, thus providing another axiom scheme for matroids.

Ordering the blocks of a design

Megan Dewar

(joint work with Brett Stevens)

MSC2000: 05B05, 05B07

The study of the presence or absence of configurations among consecutive blocks in an ordering of the blocks of a design was initiated by M. Cohen and C. Colbourn in 2003 [1]. An (n, l)-configuration is a set system with n elements and l blocks in which every element is contained in at least one block. Let $\mathcal{C}$ be a set of configurations, each consisting of l blocks. A $\mathcal{C}$-ordering of the blocks of a design is an ordering such that every l consecutive blocks form a configuration isomorphic to one of those in $\mathcal{C}$.

In this talk we will discuss the possibility of listing all blocks of a design such that every consecutive pair of blocks intersects in exactly one point and any set of three consecutive blocks in the list has an empty intersection. This is a $\mathcal{C}$-ordering where $\mathcal{C}$ is the set of configurations consisting of the path and the triangle but not containing the claw. We prove that every cyclic $\mathrm{TS}(v, \lambda)$ is $\mathcal{C}$-orderable. The proof method is constructive and therefore, similar techniques can be applied to BIBDs with $k > 3$ and to PBDs.

Reference.

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Surgeries on latin trades

Aleš Drápal

MSC2000: 05B15

The latin trade is usually defined as a partial latin square to which there exists a mate with the property that (1) the same cells are filled in both mates, (2) no cell is filled in both of the mates in the same way and (3) both the rows and columns are balanced (a row is balanced if the sets of elements appearing in the row are the same in both mates). A pair (K, K') consisting of a latin trade and its mate will be called a latin bitrade. For each row consider the permutation that moves a cell with entry e in K to the cell with entry e in K'. Similar permutations can be formed for columns and for transversals induced by the entry values. Each latin bitrade can be associated with an oriented combinatorial surface in which the cycles of the permutations form one kind of faces, while the other kind of faces are the triangles that are obtained from every triple of cycles that pairwise intersect each other (one of the three cycles is induced by a row, another by a column and the last by an entry value) [2, 3]. Call a latin bitrade *spherical*, if the genus of the associated surface is equal to 0. We shall report two constructions, one of which allows to obtain all spherical latin bitrades from those with four entry cells, while the other one yields latin bitrades of arbitrary genus starting from the spherical bitrades. These are not the only ways how one can construct new latin bitrades from simpler ones [1]. However, the presented constructions seem to be the first ones with clear geometric interpretation.

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One small step towards proving the PPC

J.E. Dunbar

(joint work with M. Frick)

MSC2000: 05C38

The order of a longest path in a graph G, called its *detour order*, is denoted by $\tau(G)$. If (a, b) is a pair of positive integers, a partition (A, B) of the vertex set of a graph G is called an (a, b)*-partition* if $\tau(G\langle A\rangle) \leq a$ and $\tau(G\langle B\rangle) \leq b$. If a graph G has an (a, b)-partition for every pair of positive integers (a, b) such that $a + b = \tau(G)$, then G is called τ*-partitionable.*

The Path Partition Conjecture (PPC) is the following: Every graph is τ-partitionable.

We show that in order to prove the PPC is true, it is sufficient to show that all non-separable graphs are τ-partitionable.

Upper bounds on planarization of bounded degree graphs

Keith Edwards

(joint work with Graham Farr)

MSC2000: 05C99, 05C10

It is known that every graph of maximum degree 3 can be planarized (i.e. made planar) by removing at most $\frac{1}{4}$ of its vertices, and that the proportion $\frac{1}{4}$ is the least for which this is true.

When the maximum degree is some $d \geq 4$, we know upper and lower bounds on the corresponding minimum fraction of the vertices whose removal can be guaranteed to planarize the graph, but the precise minimum fraction is not known.

We will describe some progress on finding better upper bounds.

Existence of disjoint cycles containing specified vertices

Y. Egawa

MSC2000: 05C38

In 1997, it was implicitly conjectured by several people that if G is a graph with $\sigma_2(G) \geq |V(G)|$ and $\delta(G) \geq k+1$, $v_1, \cdots, v_k \in V(G)$, and if there exist vertex-disjoint cycles $D_1, \cdots, D_k$ such that $v_i \in V(D_i)$ for each $1 \leq i \leq k$, then there exist vertex-disjoint cycles $C_1, \cdots, C_k$ such that $v_i \in V(C_i)$ for each $1 \leq i \leq k$ and $V(G) = \bigcup_{1 \leq i \leq k} V(C_i)$. The conjecture was disproved in 2002. On the other hand, the conjecture becomes true if we add the assumption that $\Sigma_{1 \leq i \leq k} |V(D_i)|$ is sufficiently large. For example, it is fairly easy to prove the following proposition.

Proposition. Let $k \geq 2$, and let G be a graph with $\sigma_2(G) \geq |V(G)|$ and $\delta(G) \geq k+1$. Let $v_1, \cdots, v_k$ be distinct vertices of G, and suppose that there exist vertex-disjoint cycles $D_1, \cdots, D_k$ such that $v_i \in V(D_i)$ for each $1 \leq i \leq k$. Suppose further that $\Sigma_{1 \leq i \leq k} |V(D_i)| \geq 4k$. Then there exist vertex-disjoint cycles $C_1, \cdots, C_k$ such that $v_i \in V(C_i)$ for each $1 \leq i \leq k$ and $V(G) = \bigcup_{1 \leq i \leq k} V(C_i)$.

The lower bound $4k$ on $\Sigma_{1 \leq i \leq k} |V(D_i)|$ in the assumption of the Proposition seems far from best possible. In fact, we can show that there exists a constant $\epsilon > 0$ such that the conclusion of the Proposition holds even if we replace the condition that $\Sigma_{1 \leq i \leq k} |V(D_i)| \geq 4k$ by the weaker condition that $\Sigma_{1 \leq i \leq k} |V(D_i)| \geq (4 - \epsilon)k$. In this talk, I will overview recent efforts toward the determination of the best possible lower bound.

Logarithmic terraces

L. Ellison

(joint work with Ian Anderson)

MSC2000: 11A07, 05B30

Let p be an odd prime, and let x be a primitive root of p. Suppose that we write the elements of $\mathbb{Z}_{p-1}$ as $1, 2, ..., p-1$, and that, when we evaluate x^l modulo p, we always write it as one of $1, 2, ..., p-1$. Let $\mathbf{l} = (l_1, l_2, \ldots, l_{p-1})$ be a terrace for $\mathbb{Z}_{p-1}$. Then $\mathbf{l}$ is said to be a logarithmic terrace if $\mathbf{e} = (e_1, e_2, \ldots, e_{p-1})$, defined by $e_i = x^{l_i} \pmod p$, is also a terrace for $\mathbb{Z}_{p-1}$. We study properties of logarithmic terraces, in particular investigating terraces which are simultaneously logarithmic for two different primitive roots of p.

Maximal sets of unit-distant points

C. Elsholtz

(joint work with W. Klotz)

MSC2000: 51K05, 52C35

We study maximal sets of points mutually distance 1 apart. Let $\mathbb{F}$ denote a field, and $f(\mathbb{F}^n)$ be the cardinality of a maximal set in dimension n. The regular simplex shows that $f(\mathbb{R}^n) = n + 1$. For which n can this simplex be rotated such that all coordinates are rational? A full evaluation of $f(\mathbb{Q}^n)$ is given, depending only on the prime factorizations of n and $n+1$. Our results imply that for almost all even n one has $f(\mathbb{Q}^n) = n$ and for almost all odd n one has $f(\mathbb{Q}^n) = n - 1$.

This apparently geometric or algebraic question is solved by methods from number theory and design theory. We also study the case of general fields.

On the symmetric Ashkin-Teller model and Tutte-Whitney functions

G.E. Farr

MSC2000: 05C99

In this talk we describe a family of functions that generalise the usual Tutte-Whitney polynomial of a graph or matroid. These may be viewed as forming a continuum between the Tutte-Whitney polynomials of a graph (or binary matroid) and its dual. We then discuss a connection with statistical mechanics. The partition function of the symmetric Ashkin-Teller model on a graph is not a partial evaluation of the Tutte-Whitney polynomial, although two of its specialisations (the partition functions of the Ising and Potts models) are. We show that the symmetric Ashkin-Teller partition function is a partial evaluation of a generalised Tutte-Whitney function drawn from the continuum mentioned above.

Algorithmic aspects of Queen domination

H. Fernau

MSC2000: 05C69

Many papers have been devoted to the study of the following two problems on the positioning of queens on a chessboard:

1. What is the minimum number of queens that dominate an $n \times n$ chessboard?
2. In how many ways can n queens be positioned on an $n \times n$ board?

Observe that such problems are often posed as introductory examples for backtracking algorithms in Introduction to Programming lectures, but little seems to be known about their actual computational complexity.

In this talk, we exhibit progress on the computation of the queen domination number from the viewpoint of parameterized complexity, using the natural parameter k upperbounding the number of queens we allow to be positioned on the board.

To this end, we first show a kernelization result. Then, we compare two natural approaches that easily beat the naive backtracking algorithm: (a) dynamic programming on subsets and (b) a tree decomposition based approach. Both approaches allow for $O(c^k + n)$ algorithms, where $c = 225$ with method (b) gives the better result.

A sum labelling for the flower $f_{q,p}$

H. Fernau

(joint work with J. Ryan, K.A. Sugeng)

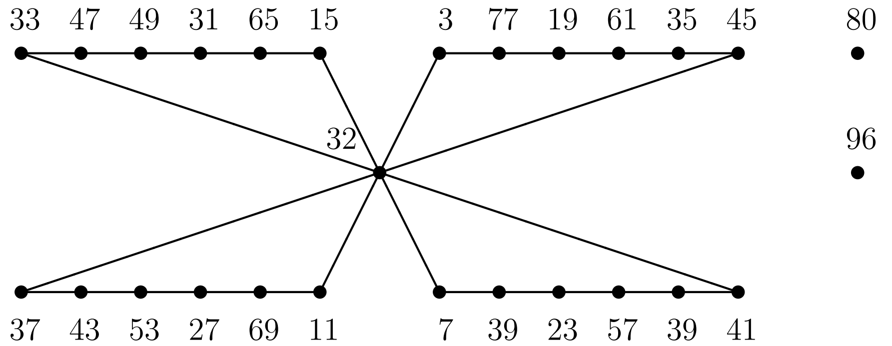
MSC2000: 05C78

A sum labeling is a mapping λ from the vertices of G into the positive integers such that for any two vertices $u, v \in V(G)$ with labels $\lambda(u)$ and $\lambda(v)$ respectively, (uv) is an edge if and only if $\lambda(u) + \lambda(v)$ is the label of another vertex in $V(G)$. Any graph supporting such a labeling is called a *sum graph*. Sum graphs are necessarily disconnected so in order to sum label a connected graph it became necessary to add (as a disjoint union) a further component. By convention this disconnected component is a set of isolated vertices known as *isolates* and the labeling scheme that requires the fewest isolates is termed *optimal*. The number of isolates required for a graph to support a sum labelling is known as the *sum number* of the graph.

Sum labeling of graphs was introduced by Harary in 1990 and since that time the problem of finding an optimal labeling for a family of graphs has been shown to be difficult, even for fairly simple graphs.

The *generalised friendship graph* $f_{q,p}$ is a collection of p cycles (all of order q), meeting at a common vertex. Note that $f_{3,n}$ is usually known as a friendship graph. The generalised friendship graph is, because of its shape, also referred to as a *flower*. In this nomenclature the cycles are referred to as petals. We will present the following result:

The generalized friendship graph $F_{q,p}$ has sum number 2.



An optimal sum numbering for $f_{7,4}$.

On well-covered planar triangulations

Art Finbow

(joint work with B.L. Hartnell, R. Nowakowski and Michael D. Plummer)

MSC2000: 05C69, 05C10

A graph G is said to be *well-covered* if every maximal independent set of vertices has the same cardinality. A planar (simple) graph in which each face is a triangle is called a *triangulation*. The aim of this project is to characterize the planar well-covered triangulations. At this point we have completed the 4- and 5-connected cases.

6-sparse Steiner triple systems

A.D. Forbes

(joint work with M.J. Grannell and T.S. Griggs)

MSC2000: 05B07

A *Steiner triple system* of order v (STS(v)), is a pair $(V, \mathcal{B})$ where V is a set of v points and $\mathcal{B}$ is a set of triples, also called *blocks*, such that a pair of distinct points occurs in precisely one triple. A *configuration* is a finite set of triples where a pair of points occurs at most once.

For $k \geq 4$, a Steiner triple system S of order v is called *k-sparse* if for $4 \leq n \leq k$, every configuration in S of n blocks spans at least $n+3$ points. The terminology originates from Erdős, who conjectured that for every integer $k \geq 4$, there exists $v_0(k)$ such that if $v > v_0(k)$ and v is admissible (that is, $v \equiv 1$ or 3 (mod 6)), then there exists a k-sparse STS(v).

The *Pasch* configuration, $\{012, 034, 135, 245\}$, is the only case where a configuration of 4 blocks has less than 7 points. Thus an STS(v) is 4-sparse if and only if it is *anti-Pasch*. The resolution of the anti-Pasch problem and therefore of the Erdős conjecture for $k = 4$ was established in a series of papers: Brouwer (1977), Ling, Colbourn, Grannell and Griggs (2000), and, finally, Grannell, Griggs and Whitehead (2000). There exists an anti-Pasch STS(v) for all admissible v except 7 and 13.

The *mitre*, $\{012, 034, 135, 236, 456\}$ is the only anti-Pasch configuration of 5 blocks which has less than 8 points, and therefore an STS(v) is 5-sparse if and only if it is both anti-Pasch and anti-mitre. Some progress has been made with 5-sparse STS(v)s; it is now known that such systems exist for $v \equiv$ 1, 19 (mod 54) except possibly $v = 109$, and for some other sporadic v (Ling (1997), Fujiwara (2005)).

Nothing was previously known about the next case, the subject of this talk. Here we will take the initial steps towards the Erdős conjecture for $k = 6$ by establishing the existence of 6-sparse STS(v)s for infinitely many v.

A new perspective on the Path Partition Conjecture

M. Frick

(joint work with C. Whitehead and I. Schiermeyer)

MSC2000: 05C38

The order of a longest path in a graph G is denoted by $\tau(G)$. If the difference between the order of G and $\tau(G)$ equals p, we say that G is *p-deficient.* A 0-deficient graph is called *traceable.* The following conjecture, which was formulated in 1981 but has not yet been settled, is referred to as the Path Partition Conjecture (PPC):

PPC: If G is any graph and (a, b) any pair of positive integers such that

$$a + b = \tau(G), \tag{1}$$

then G has a vertex partition (A, B) such that

$$\tau(\langle A \rangle) \leq a \text{ and } \tau(\langle B \rangle) \leq b. \tag{2}$$

We have proved that for each $p \geq 0$ there exist at most a finite number of p-deficient graphs satisfying (1) that do not satisfy (2).

Any vertex partition (A, B) of a graph satisfying (2) is called an *(a, b)-partition.* If the PPC were true, it would be "best possible" in the sense that if the condition (1) is weakened to $a + b = \tau(G) - 1$, we cannot guarantee that G has an (a, b)-partition. For example, if G is the complete graph K_{a+b+1} then $a + b = \tau(G) - 1$ but G has no (a, b)-partition. We have also constructed noncomplete traceable graphs with this property but we do not know whether nontraceable ones exist. It might well be that a stronger result than that conjectured in the PPC is true for non-traceable graphs. These considerations motivate the following definition.

Definition. The *path partition function* $f : \mathbb{Z}^+ \cup \{0\} \to \mathbb{Z}$ is defined by: $f(p)$ is the greatest integer for which every p-deficient graph G has an (a, b)-partition for every pair of positive integers (a, b) such that $a+b = \tau(G)-f(p)$.

The PPC is equivalent to the conjecture that $f(p) \geq 0$ for all $p \geq 0$.

We show that $-p \leq f(p) \leq 1$ for all $p \geq 0$. Moreover, $f(0) = 0$, $f(1) = f(2) = 1$ and $0 \leq f(3) \leq 1$.

Long cycles passing through a linear forest

J. Fujisawa

(joint work with T. Yamashita)

MSC2000: 05C38, 05C45

In 2001, Hu et al. proved a theorem which shows the existence of a long cycle passing through a linear forest, using a degree condition which considers the average degree of $k+1$ independent vertices. Recently, it is proved that we don't need to consider all the vertices, and we can guarantee the length of the cycle by the degree sum of two vertices of high degree in $k+1$ independent vertices. In the talk I will present this new result, and I will also mention a related conjecture.

Constructions for cyclic 4- and 5-sparse Steiner triple systems

Y. Fujiwara

MSC2000: 05B07

A *Steiner triple system* of *order* v, briefly STS(v), is an ordered pair $(V, \mathcal{B})$, where V is a finite set of v elements called *points*, and $\mathcal{B}$ is a set of 3-element subsets of V called *blocks*, such that each unordered pair of distinct elements of V is contained in exactly one block of $\mathcal{B}$. An STS(v) is said to be r*-sparse* if it has no set of i blocks whose union contains precisely $i+2$ points for $2 \leq i \leq r$. An STS(v) is said to be *cyclic* if its automorphism group contains a cyclic group of order v as a subgroup acting on V.

In this talk, we consider the existence problem on Steiner triple systems which are both cyclic and r-sparse. Several recursive constructions for cyclic r-sparse STSs with $r = 4, 5$ are developed.

Structure and enumeration of toroidal and projective-planar graphs with no $K_{3,3}$'s

A.V. Gagarin

(joint work with P. Leroux and G. Labelle)

MSC2000: 05C10, 05C30, 68R10

By Kuratowski's theorem, a graph G is non-planar if and only if it contains a subdivision of K_5 or $K_{3,3}$. A graph G does not contain a $K_{3,3}$-subdivision if and only if it does not contain a $K_{3,3}$-minor. Therefore such a graph is called a graph *with no* $K_{3,3}$*'s*. The graphs with no $K_{3,3}$'s can be described recursively in terms of K_5's and 2-connected planar graphs.

We provide structure theorems for toroidal and projective-planar graphs with no $K_{3,3}$'s in terms of 2-pole planar networks substituted for the edges of canonically defined non-planar graphs. These non-planar graphs are respectively called *toroidal cores* and *projective-planar cores*. The decompositions imply algorithms to detect toroidal and projective-planar graphs with no $K_{3,3}$'s. The algorithms can be implemented to run in linear time.

A proper use of mixed generating functions with an edge counter is described in detail for the operation of substitution of 2-pole networks into the edges of a graph. As a result, we count labelled 2-connected toroidal and projective-planar graphs with no $K_{3,3}$'s, and labelled 2-connected homeomorphically irreducible planar, toroidal and projective-planar graphs with no $K_{3,3}$'s. We are currently working on the unlabelled enumeration of these graphs, and have already counted the isomorphism classes of toroidal cores.

Optimal restricted connectivity and superconnectivity in graphs with small diameter

P. García–Vázquez

(joint work with C. Balbuena, M. Cera, A. Diánez and X. Marcote)

MSC2000: 05C35, 05C40

For a connected graph G, the restricted connectivity $\kappa'(G)$ is defined as the minimum cardinality of a vertex-cut over all vertex-cuts X such that no vertex u has all its neighbors in X; the superconnectivity $\kappa_1(G)$ is defined similarly, this time considering only vertices u in $G-X$, hence $\kappa_1(G) \leq \kappa'(G)$. The minimum edge-degree of G is $\xi(G) = \min\{d(u)+d(v)-2 : uv \in E(G)\}$, $d(u)$ standing for the degree of a vertex u. A graph G is said to be κ'-optimal if $\kappa'(G) = \xi(G)$, and optimally superconnected if $\delta(G) < \kappa_1(G) = \kappa'(G) = \xi(G)$, $\delta(G)$ being the minimum degree of G. In this paper, several sufficient conditions yielding $\kappa_1(G) \geq \xi(G)$ are given, guaranteeing optimal superconnectivity $\kappa_1(G) = \kappa'(G) = \xi(G)$ under some additional constraints which are based on the relationship between the diameter and the girth of G.

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Embeddings of trees and the best secretary problem

N. Georgiou

MSC2000: 06A07, 05A20

A *rooted tree*, or *tree* for short, is a partial order with a maximum element, called the root, whose Hasse diagram is a tree. A *binary tree* is a tree such that every element has at most two lower covers. The *complete binary tree* T^n of height n, is a ranked tree with n levels, such that every element except for the leaves has exactly two lower covers. An embedding of a tree T into T^n is a map $\phi : T \to T^n$ such that $\phi(x) \leq \phi(y)$ in T^n if and only if $x \leq y$ in T.

We write $C_T(n)$ for the total number of embeddings of T into T^n, and write $A_T(n)$ for the number of those that map the root of T to the root of T^n. Kubicki, Lehel and Morayne have proved that, for fixed n, if T_1, T_2 are binary trees with T_1 a subposet of T_2, then

$$\frac{A_{T_1}(n)}{C_{T_1}(n)} \leq \frac{A_{T_2}(n)}{C_{T_2}(n)}.$$

They also conjectured that the inequality holds for arbitrary trees T_1, T_2 with T_1 a subposet of T_2.

We disprove their conjecture, giving a counterexample to the inequality. As a consequence, we have the counter-intuitive result that, in a partial-order version of the best secretary problem, where candidates are ordered as a complete binary tree, the probability of the best-so-far candidate being the best-of-all candidate is not increasing in the number of candidates already interviewed. This contrasts with the total-order version of the problem.

Hexagon Biquadrangle systems

Lucia Gionfriddo

MSC2000: 05B05, 05B30

A hexagon biquadrangle system of order n and index ρ [$HBQS_\rho(n)$] is a pair (X, H), where X is a finite set of n vertices and H is a collection of edge disjoint hexagon biquadrangles (called *blocks*) which partitions the edge set of ρK_n, with vertex set X. *A hexagon biquadrangle system* is said to be a 4-*nesting* [$N(4) - HBQS$] if the collection of all the 4-cycles contained in the hexagon biquadrangles form a μ-fold 4-cycle system. It is said to be a 6-*nesting* [$N(6) - HBQS$] if the collection of 6-cycles contained in the hexagon biquadrangles is a λ-fold 6-cycle system. It is said to be a $(4, 6)$-*nesting*, briefly a $N(4, 6) - HBQS$, if it is both 4-*nesting* and a 6-*nesting*.

It is said to be a $(4^2, 6)$-nesting if it is $(4, 6)$-nesting and the μ-fold 4-cycle system, nested in it, is decomposable into two $\frac{\mu}{2}$-fold 4-cycle systems.

In this research we determine completely the spectrum of $N(4^2, 6) - HBQS$ for $\rho = 7h$, $\lambda = 6h$ and $\mu = 8h$, h positive integer.

All vertex-transitive locally-quasiprimitive graphs have a semiregular automorphism

Michael Giudici

(joint work with Jing Xu)

MSC2000: 20B25, 20B05

A *semiregular* permutation is a permutation whose cycles all have the same size. The polycirculant conjecture states that every transitive 2-closed permutation group contains a semiregular element. The full automorphism group of a graph is 2-closed but not every 2-closed permutation group is the full automorphism group of some graph. In this paper we make substantial progress on the polycirculant conjecture by proving that every vertex-transitive, locally quasiprimitive graph has a semiregular automorphism. The main ingredient of the proof is the determination of all biquasiprimitive permutation groups which do not contain a semiregular element.

Approximate counting: Independent sets and Ferromagnetic Ising

Leslie Ann Goldberg

(joint work with Mark Jerrum)

MSC2000: 68W20, 05C85, 68Q15

We start by reviewing the computational difficulty of combinatorial counting problems. Even though the complexity of many natural problems is unresolved (for example, we do not know whether the number of independent sets in a bipartite graph can be efficiently approximated) we do know some computational equivalences between problems. In this work we consider a particular counting problem arising in statistical physics: namely, approximating the partition function of the ferromagnetic Ising model with varying interaction energies and local external magnetic fields. Jerrum and Sinclair provided an efficient approximation algorithm for the case in which the system is consistent, in the sense that the local external fields all favour the same spin. We show that the general problem is equivalent in complexity to the independent set problem mentioned above. This implies that it is complete in a logically-defined subclass of #P previously studied by Dyer, Goldberg, Greenhill and Jerrum. In contrast, we show that approximating the partition function of the q-state Potts model with $q > 2$ is as hard as approximately solving any counting problem in #P — for example, it is as hard as approximately counting independent sets in an arbitrary graph.

A flaw in the use of minimal defining sets for secret sharing schemes

M.J. Grannell

(joint work with T.S. Griggs and A.P. Street)

MSC2000: 94A62, 05B05

A defining set for a $t-(v,k,\lambda)$ design is a collection of k-tuples which is contained in a unique design with the given parameters. A minimal defining set for a $t-(v,k,\lambda)$ design is a defining set for the design, no proper sub-collection of which is a defining set. However, we show that in some cases it is possible to reconstruct a $t-(v,k,\lambda)$ block design $\mathcal{D}$ uniquely from a proper sub-collection $\mathcal{S}^*$ of a minimal defining set $\mathcal{S}$ for $\mathcal{D}$, given only the additional information that $\mathcal{S}^*$ is indeed a sub-collection of some minimal defining set for a $t-(v,k,\lambda)$ design. This surprising result has implications for the use of minimal defining sets in secret sharing schemes.

Bounds on the generalised acyclic chromatic numbers of bounded degree graphs

Catherine Greenhill

(joint work with Oleg Pikhurko)

MSC2000: 05C15, 05C38

The acyclic chromatic number $A(G)$ of a graph G is the minimum number of colours required to properly colour the vertices of G such that every cycle has more than 2 colours. The quantity $A(G)$ was introduced by Grünbaum in 1973, in the context of planar graphs. A similar definition for edge colourings leads to the acyclic edge chromatic number $A'(G)$ of G. These numbers can be generalised as follows. Fix $r \geq 3$ and let $A_r(G)$ (respectively, $A'_r(G)$) be the minimum number of colours required to properly colour the vertices (respectively, edges) of G such that every cycle C in the graph G receives $\min\{|C|, r\}$ colours. So $A(G) = A_3(G)$ and $A'(G) = A'_3(G)$.

We give upper bounds for the generalised acyclic chromatic number and generalised acyclic edge chromatic number of graphs with maximum degree d, as a function of d. We also produce examples of graphs where these bounds are of the correct order.

Steiner triple systems and existentially closed graphs

T.S. Griggs

(joint work with A.D. Forbes and M.J. Grannell)

MSC2000: 05C99, 05B07

A graph is said to be *n-existentially closed*, or *n-e.c.*, if for every n-element subset S of the vertex set, and every subset T of S, there exists a vertex $x \notin S$ which is adjacent to every vertex in T, and is not adjacent to any vertex in $S \setminus T$. In 1963, Erdős and Rényi proved that for any fixed value of n, almost all graphs are n-existentially closed. But relatively few specific examples are known for $n \geq 2$.

Recent research has focused on strongly regular graphs with existentially closed properties. Baker, Bonato and Brown (2003) constructed 3-e.c. graphs from affine planes and Bonato, Holzmann and Kharagani (2001) studied 3-e.c. graphs obtained from Hadamard matrices.

In this talk, I will explore the existentially closed properties of the block intersection graphs of Steiner triple systems. These naturally relate to questions concerning configurations. We are able to prove that the block intersection graph of every Steiner triple system except the unique systems on 7 and 9 points is 2-e.c. and obtain a characterization of those Steiner triple systems whose block intersection graphs are 3-e.c.

This leads to the interesting result that there are at most two orders of Steiner triple system, namely 19 and 21, for which the block intersection graph can be 3-existentially closed. But they do exist and we identify two such systems on 19 points. The case of 21 points remains elusive.

On the number of power-free words in two and three letters

Uwe Grimm

MSC2000: 68R15, 05A15

An interesting problem in the combinatorics of words concerns the number of words that avoid certain powers. The best studied example is the set of ternary square-free words. In this talk, we discuss methods that lead to improved lower and upper bounds. Improved lower bounds can be obtained by a suitable generalisation of Brinkhuis triples. Essentially this relies on identifying a selection of square-free morphisms that are mutually compatible, such that any combination of the morphisms preserves the square-freeness of words. Such a set of morphisms then provides an exponential lower bound. This approach has been applied to improve the lower bound of the number of square-free ternary words of length n from $2^{n/17}$, which was derived using a 'traditional' Brinkhuis triple, to $65^{n/40}$ [1]. This bound was subsequently verified independently, and, employing the same method, further improved to $110^{n/42}$ [2]. An improved upper bound has been obtained by calculating the full generating function of length-ℓ square-free words for $\ell \leq 24$ [3].
We also report briefly on some interesting results concerning power-free binary words [4,5], and on preliminary results on the number of binary cube-free words.

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Initial results from a study of probability curves for shortest arcs in optimal ATSP tours with application to heuristic performance

Vic Grout

MSC2000: 90C27, 90C59

Define the following *Asymmetric Traveling Salesman Problem* (*ATSP*) classes on n cities:

- The *k-Discrete* TSP (*kDTSP*), D_n^k, with arc costs drawn randomly (uniformly) from $\{0,1, \ldots ,k\text{-}1\}$. A special case is the *Binary* TSP (*BTSP*), $B_n = D_n^2$, with arc costs in $\{0,1\}$.
- The *Permuted* TSP (*PTSP*), P_n, with arc costs drawn uniquely (permuted) from $\{0,1, \ldots\ n(n\text{-}1)\text{-}1\}$.
- The *Uniform* TSP (*UTSP*), U_n, with arc costs drawn from some continuous interval with uniform probability.
- The *Normal* TSP (*NTSP*), $N_n^{\mu\sigma}$, with arc costs drawn from some continuous interval according to a normal distribution with mean μ and variance σ^2.

Define $\rho(i,A_n)$, $\rho{:}\{1,2, \ldots\ n(n\text{-}1)\}\times A_n \to [0,1]$, for a given ATSP class A_n, to be the probability that the i^{th}-shortest arc appears in the optimal tour for any instance of A_n. Define $\eta(j,A_n)$, $\eta{:}\{1,2, \ldots\ n\text{-}1\}\times A_n \to [0,1]$, for a given ATSP class A_n, to be the probability that, for any city, the arc to its j^{th}-nearest neighbour appears in the optimal tour for any instance of A_n.

Some simple results may be conjectured or derived by exhaustion, such as:

$\rho(i,A_n) \geq \quad \rho(i+1,A_n)$for all classes A_n,

$\rho(1,\ TRIAL\ RESTRICTION\ ) = \rho(1,B_n) = 1$

$\rho(1,D_2^3) = 1$

$\rho(1,P_3) = 0.8$

with corresponding results for η. However, for larger n and i (and j), this approach is not viable.

The paper reports on initial results from large-scale empirical testing to determine these probability curves and attempts to relate values obtained for ρ and η to degrees of accuracy for various greedy and greedy-type heuristics for different ATSP classes.

A $\Delta + 4$ bound on the total chromatic number for graphs with chromatic number on the order of $\sqrt{\Delta / \log \Delta}$

R. Häggkvist

MSC2000: 05C15

The total chromatic number for a graph G is the least number of colours needed to color the vertices and edges of G, such that no adjacent vertices, no adjacent edges and no incident vertices and edges receive the same colour. It has been conjectured by Behzad (1965) and Vizing (1968) that the total chromatic number is at most $\Delta + 2$, where $\Delta = \Delta(G)$ is the maximum degree of G. The upper bound $\Delta + 10^{26}$ was established by Molloy & Reed (1998). The method used in the present result differs from that of Molloy & Reed in that, for instance, it does not require the existence of a special vertex colouring with certain properties. Instead, the key result used in the proof of the assertion in the title is that one may start from *any* proper vertex colouring, with the single requirement that not too many colours are used.

Another result used in the proof is that every m-regular graph has a spanning bipartite subgraph H with vertices of degree s or $s-1$ for every $s \leq \lfloor \frac{1}{4}\sqrt{m/\log 3m} \rfloor$. It would be of interest to establish a best possible version of this proposition. In particular, it might be that it holds for s as large as $\frac{m}{4}$, say.

$(r, r+1)$-factorizations of multigraphs with high minimum degree

A.J.W. Hilton

MSC2000: 05C15, 05C70

For $r \geq 0$, an $(r, r+1)$-*factor* of a multigraph G is a spanning subgraph of G each of the degrees of which is either r or $r+1$. An $(r, r+1)$-*factorization* of G is a decomposition of G into edge-disjoint $(r, r+1)$-factors.

Let $r \geq 0$, $s \geq 0$ and let $\psi(r, s)$ be the least integer such that, if G is a multigraph (without loops) with minimum degree $\delta(G) \geq \psi(r, s)$ and maximum degree $\Delta \leq \delta + s$, then G has an $(r, r+1)$-factorization. We show that $\psi(r, s)$ exists for all r, s, and give the upper bound:

$$\psi(r, s) \leq 4(4r^2 + 6r + 5)\left(s + 8(4r^2 + 6r + 5)\right).$$

Semi-total graph colourings, the beta parameter, and total chromatic number

Fred Holroyd

(joint work with Jini Williams)

MSC2000: 05C15

A *semi-total colouring* of a graph G with maximum degree Δ uses $\Delta + 1$ colours, and has the properties of a total colouring *except* that adjacent vertices need not have distinct colours.

Given such a colouring, μ, of G, a *beta edge* of G is an edge incident with two similarly coloured vertices, and $\beta_\mu(G)$ is the number of beta edges with respect to μ. Finally, $\beta(G) = \min\{\beta_\mu(G) : \mu \text{ is a semi-total colouring of } G\}$.

A graph G is *nearly of type 1* if the deletion of just one edge not contained in a triangle reduces the total chromatic number of G to $\Delta + 1$. We derive a bound on $\beta(G)$ for such a graph, that is log-linear in Δ.

Multiple chromatic numbers of some Kneser graphs

Fred Holroyd

(joint work with Andonis Yannakopolous)

MSC2000: 05C15

The *Kneser graph* $K(m,n)$ (where $m > 2n$) has the n-sets of an m-set as its vertices, two vertices being adjacent whenever they are disjoint as sets. The *kth chromatic number* of any graph G is the least integer t such that the vertices can be assigned k-subsets of $\{1,\dots,t\}$ with adjacent vertices always receiving disjoint sets. Saul Stahl has conjectured that, if $k = qn - r$ where $q \geq 1$ and $0 \leq r \leq n$, then the kth chromatic number of $K(m,n)$ is $qm - 2r$. This is easily verified when $r = 0$; Stahl has also established its validity when $m = 2n+1$ and when $n = 2, 3$.

We establish the validity of the conjecture in the following further classes of cases:

(i) $2 \leq \frac{m}{n} < 2 + \frac{1}{r}$;

(ii) $4 \leq n \leq 6$ and $1 \leq r \leq 2$;

(iii) $7 \leq n \leq 11$ and $r = 1$;

(iv) $(n,r,m) = (7,2,18), (12,1,37), (12,1,38)$ or $(13,1,40)$.

General neighbour-distinguishing index of a graph

M. Horňák

(joint work with E. Győri, C. Palmer and M. Woźniak)

MSC2000: 05C15

It is proved that edges of a graph G with no component K_2 can be coloured using at most $2\lceil\log_2\chi(G)\rceil + 1$ colours so that any two adjacent vertices have distinct sets of colours of their incident edges.

Bounds on optimal edit metric codes

S.K. Houghten

(joint work with D. Ashlock and J. Campbell)

MSC2000: 94B60, 94B65

The *edit distance* between two strings is the minimal number of substitutions, deletions, or insertions required to transform one string into another. An error correcting code over the edit metric includes features from deletion-correcting codes as well as the more traditional codes defined using Hamming distance. Applications of edit metric codes include the creation of robust tags over the DNA alphabet.

While codes over the edit metric are analogous to similar codes over the Hamming metric, little of the beautiful theory survives. The *block structure* of a word is its partition into maximal subwords composed of a single character. The size of a sphere about a word in the edit metric is heavily dependent on the block structure of the word, creating a substantial divergence from the theory for the Hamming metric.

This paper explores the theory underlying edit metric codes for small alphabets. An *optimal code* is a code of maximal size for a given length and minimum distance. We provide tables of bounds on code sizes for edit codes with short length and small alphabets. We present several heuristics for constructing codes.

Variable changes for generalized power series

I-Chiau Huang

MSC2000: 05E99, 05A19, 13F25

Rings of formal power series and the operation of equating coefficients provide an algebraic foundation for the method of generating functions. However, without employing the notion of Kähler differentials, the effect of variable changes is not transparent. With meromorphic differentials, contour integrations give an alternative way to take coefficients. While Jacobians occurring in variable changes fit perfectly in such an analytic framework, divergence of sequences may cause discomfort. Removing unnecessary analytic restrictions, the author arrives at certain cohomology classes of separated differentials [1]. The process of integration is replaced by residue maps, which plan a significant role in Grothendieck duality theory. The new algebraic foundation interprets naturally Lagrange inversion formulae [2] and inverse relations [4].

The formalism of cohomology residues is simple. For a power series in variables $X_1, \cdots, X_n$,

$$\operatorname{res}\begin{bmatrix} \varphi dX_1 \cdots dX_n \\ X_1, \cdot, X_n \end{bmatrix} = \text{ constant coefficient of } \varphi.$$

Although working well on wide range of problems in combinatorial analysis [3], the derivation dX_i^{-1} of the inverse of a variable X_i is not defined. In the talk, we work on a field $\kappa[[T^{\mathcal{G}}]]$ of generalized power series, where $\mathcal{G}$ is a totally ordered Abelian group. The definition of derivation is extend to dX_i^{-1}. The logarithmic analogue

$$\operatorname{res}\begin{bmatrix} \varphi d\log X_1 \cdots d\log X_n \\ \log X_1, \cdots, \log X_n \end{bmatrix}$$

of residues is defined, even for a field of positive characteristic. The well-known formula of Jacobi and various proofs of Dyson's conjecture are interpreted naturally.

1. I-C. Huang. Applications of residues to combinatorial identities. *Proc. Amer. Math. Soc.*, 125(4):1011–1017, 1997.
2. I-C. Huang. Reversion of power series by residues. *Comm. Algebra*, 26(3):803–812, 1998.
3. I-C. Huang. Residue methods in combinatorial analysis. In *Local Cohomology and its Applications*, volume 226 of *Lecture Notes in Pure and Appl. Math.*, pages 255–342. Marcel Dekker, 2001.
4. I-C. Huang. Inverse relations and Schauder bases. *J. Combin. Theory Ser. A*, 97(2):203–224, 2002.

The Doyen-Wilson Theorem for Extended Directed Triple Systems

Wen-Chung Huang

MSC2000: 05B07

An extended directed triple system of the order n, EDTS(n), is a pair (V, B), where B is a collection of ordered triples from a n-set V (each ordered triple may have repeated elements) such that every ordered pair of elements of V, not necessarily distinct, is contained in exactly one ordered triple of B.

In this paper, it is shown that every extended directed triple system of the order v can be embedded in an extended directed triple system of the order n for all $n \geq 2v$. This produces a generalization of the Doyen-Wilson theorem for extended directed triple systems.

Frequency Permutation Arrays

S. Huczynska

(joint work with G. Mullen)

MSC2000: 94A29, 94A05

Motivated by recent interest in permutation arrays, we introduce and investigate the more general concept of frequency permutation arrays (FPAs). An FPA of length $n = m\lambda$ and distance d is a set T of multipermutations on a multiset of m symbols, each repeated with frequency λ, such that the Hamming distance between any distinct $x, y \in T$ is at least d. Such arrays have potential applications in powerline communication. We establish basic properties of FPAs, and provide direct constructions for FPAs using a range of combinatorial objects. We also provide recursive constructions, and give bounds for the maximum size of such arrays.

Unique realizations of graphs

Bill Jackson

(joint work with Tibor Jordán)

MSC2000: 05C10, 05C62, 05C75

A d-dimensional *framework* is a straight line realization of a graph G in $\mathbb{R}^d$. We consider *generic* frameworks, in which the set of co-ordinates of all the vertices of G is algebraically independent over the rationals. Two frameworks for G are *equivalent* if corresponding edges in the two frameworks have the same length. A framework is a *unique realization* of G in $\mathbb{R}^d$ if every equivalent framework can be obtained from it by an isometry of $\mathbb{R}^d$. Bruce Hendrickson proved that if G has a unique realization in $\mathbb{R}^d$ then G is $(d+1)$-connected and redundantly rigid. He conjectured that every realization of a $(d+1)$-connected and redundantly rigid graph in $\mathbb{R}^d$ is unique. This conjecture is true for $d = 1$ but was disproved by Robert Connelly for $d \geq 3$. We resolve the remaining open case by showing that Hendrickson's conjecture is true for $d = 2$. As a corollary we deduce that every realization of a 6-connected graph as a 2-dimensional generic framework is a unique realization. Our proof is based on a new inductive characterization of 3-connected graphs whose rigidity matroid is connected.

A degree constraint for uniquely Hamiltonian graphs

A. Jamshed

(joint work with Sarmad Abbasi)

MSC2000: 05D40, 05C45

This paper is concerned with uniquely Hamiltonian graphs. As the name suggests, a graph is called uniquely Hamiltonian if it contains exactly one Hamilton cycle. In the *Fifth British Combinatorial Conference (1975)*, J. Sheehan asked if every uniquely Hamiltonian graph contains a vertex of low degree. J. A. Bondy and B. Jackson proved that every uniquely Hamiltonian graph contains a vertex of degree at most $c' \log_2 4n + 3$ where $c' = (2 - \log_2 3)^{-1} \approx 2.41$. This result improves the naïve argument based on Dirac's Theorem that uniquely Hamiltonian graphs must have a vertex of degree at most $\frac{n}{2} + 1$ and an earlier observation of B. Jackson and R. W. Whitty that uniquely Hamiltonian graphs must have a vertex of degree at most $\frac{n+9}{4}$. We prove that if $G = (V, E)$ is uniquely Hamiltonian then

$$\sum_{v \in V} \left(\frac{2}{3}\right)^{d(v) - \#(G)} \geq 1.$$

Where $\#(G) = 1$ if G has even number of vertices and 2 if G has odd number of vertices. It follows that every n-vertex uniquely Hamiltonian graph contains a vertex whose degree is at most $c \log_2 n + 2$ where $c = (\log_2 3 - 1)^{-1} \approx 1.71$ thereby improving the bound given by J. A. Bondy and B. Jackson.

This method also gives some useful information about uniquely Hamiltonian graphs. We can also say something about the location of the small degree vertices. For example, one can show that every uniquely Hamiltonian graph either contains two vertices of small degree that are adjacent in the Hamilton cycle or it contains reasonable number of vertices that have small degree.

Two remarks concerning balanced matroids

Mark Jerrum

MSC2000: 05B35, 05A15, 51E10, 68Q17

The property of balance (in the sense of Feder and Mihail) is investigated in the context of paving matroids. The following examples are exhibited: (a) a class of "sparse" paving matroids that are balanced, but at the same time rich enough combinatorially to permit the encoding of hard counting problems; and (b) a paving matroid that is not balanced. The computational significance of (a) is the following. As a consequence of balance, there is an efficient algorithm for approximating the number of bases of a sparse paving matroid within specified relative error. On the other hand, determining the number of bases exactly is likely to be computationally intractable.

Connectedness of graphs of vertex-colourings

Matthew Johnson

(joint work with Luis Cereceda and Jan van den Heuvel)

MSC2000: 05C15, 05C40, 05C85

For a graph G, the k-colour graph, $\mathcal{C}_k(G)$, has as its vertex set the proper vertex k-colourings of G; two colourings in the vertex set of $\mathcal{C}_k(G)$ are adjacent if they differ on precisely one vertex of G.

We will show that

- for every 3-chromatic graph G, $\mathcal{C}_3(G)$ is not connected,
- for all $k \geq 4$, there exist k-chromatic graphs whose k-colour graph is connected, and
- for all $2 \leq p \leq k$, there are p-chromatic graphs whose k-colour graph is not connected.

We will also show how to recognize, in polynomial time, whether, for any 3-colourable graph G, two vertices of $\mathcal{C}_3(G)$ belong to the same connected component, and how to find a path between them if they do.

Universal cycles for permutations and other combinatorial families

Robert Johnson

MSC2000: 05A99

A de Bruijn cycle of order n is a sequence in $\{0,1\}^{2^n}$ in which each n-tuple in $\{0,1\}^n$ occurs exactly once as a cyclic interval. In 1992, Chung, Diaconis and Graham introduced the notion of a universal cycle, which generalises this idea to other combinatorial families. In this talk we describe some recent work which answers a question of these authors on universal cycles for permutations. We will also survey briefly some results and conjectures in the area.

Extremal results for rooted minor problems

L.K. Jørgensen

(joint work with K. Kawarabayashi)

MSC2000: 05C83

We consider rooted minors in graphs, i.e., graph minors containing a specified set of vertices. In particular if X is a set of k vertices in a graph G then a rooted $K_{\ell,k}(X)$ minor consists of disjoint connected subgraphs $V_1, \dots, V_\ell, W_1, \dots, W_k$ of G so that G has a $V_i - W_j$ edge for every pair i, j, and $|X \cap W_j| = 1$ for every j.

We previously used an extremal result for rooted $K_{2,4}(X)$ minors to prove an extremal result for $K_{4,4}$ minors in 4-connected graphs. With Kawarabayashi we now prove that every 4-connected graph with n vertices and at least $5n-14$ edges has a rooted $K_{3,4}(X)$ minor. We also consider rooted $K_{3,3}(X)$ minors and rooted $K_{3,2}(X)$ minors.

A graphic generalisation of Arithmetic

Delaram Kahrobaei

(joint work with K. Bhutani and B. Khan)

MSC2000: 05C99, 11U10

In this talk, we extend the classical arithmetic defined over the set of natural numbers $\mathbb{N}$, to the set of all finite directed connected multigraphs having a pair of distinguished vertices. Specifically, we introduce a model $\mathcal{F}$ on the set of such graphs, and provide an interpretation of the language of arithmetic $\mathcal{L} = \{0, 1, \leqslant, +, \times\}$ inside $\mathcal{F}$. The resulting model exhibits the property that the standard model on $\mathbb{N}$ embeds in $\mathcal{F}$ as a submodel, with the directed path of length n playing the role of the standard integer n. We will compare the theory of the larger structure $\mathcal{F}$ with classical arithmetic statements that hold in $\mathbb{N}$. For example, we explore the extent to which $\mathcal{F}$ enjoys properties like the associativity and commutativity of $+$ and $\times$, distributivity, divisibility, and order laws.

The circular chromatic index of graphs of high girth

T. Kaiser

(joint work with D. Král', R. Škrekovski and X. Zhu)

MSC2000: 05C15

A *circular ℓ-edge-coloring* of a graph G (for a real $\ell \geq 1$) is a coloring of the edges of G by the points of a circle C of circumference ℓ, such that the distance on C of the colors assigned to any two incident edges is at least 1. The *circular chromatic index* $\chi'_c(G)$ of G is the least ℓ for which G admits a circular ℓ-edge-coloring (the minimum is always attained and is a rational number).

It was conjectured by Jaeger and Swart that non-3-edge-colourable cubic bridgeless graphs have bounded girth. Although this 'Girth Conjecture' has been disproved, we show that its analogue for the circular edge-colouring 'holds' in an asymptotic sense. More generally, we prove the following Vizing-type theorem for the circular chromatic index of graphs of large girth: For each $\varepsilon > 0$ and each integer $\Delta \geq 1$, there exists a number g such that for any graph G of maximum degree Δ and girth at least g, the circular chromatic index of G is at most $\Delta + \varepsilon$.

More large sets of resolvable MTS and DTS

Qingde Kang

(joint work with Hongtao Zhao and Rongjia Xu)

MSC2000: 05B07

A *cyclic* (resp. *transitive*) *triple* on a v-set X is a set of three ordered pairs: (x, y), (y, z) and (z, x) (resp. (x, z)) of X, which is denoted by $\langle x, y, z\rangle$ or $\langle y, z, x\rangle$, or $\langle z, x, y\rangle$ (resp. (x, y, z)). An *Mendelsohn* (resp. *directed*) *triple system* of order v, denoted by $MTS(v)$ (resp. $DTS(v)$), is a pair $(X, \mathcal{B})$ where $\mathcal{B}$ is a collection of cyclic (resp. transitive) triples on X, such that each ordered pair of X occurs in exactly one triple of $\mathcal{B}$. An $MTS(v)$ (resp. $DTS(v)$) is called *resolvable* and is denoted by $RMTS(v)$ (resp. $RDTS(v)$), if its blocks can be partitioned into *parallel classes*, each containing every element of X exactly once.

A *large set* of Mendelsohn (resp. directed) triple systems of order v, denoted by $LMTS(v)$ (resp. $LDTS(v)$), is a collection $\mathcal{A}$ of $(v-2)$ $MTS(v)$s (resp. $3(v-2)$ $DTS(v)$s) based on X such that every cyclic (resp. transitive) triple from X occurs in exactly one member of $\mathcal{A}$. It is well known that an $LMTS(v)$ (an $LDTS(v)$) exists if and only if $v \equiv 0, 1 \pmod 3$ with an exception $LMTS(6)$; an $RMTS(v)$ (and $RDTS(v)$) exists if and only if $3|v$ and $v \neq 6$. The large set consisted by $RMTS(v)$ (resp. $RDTS(v)$) is denoted by $LRMTS(v)$ (resp. $LRDTS(v)$). The existence of $LRMTS(v)$ and $LRDTS(v)$ have been investigated by many scholars. By their research, $LRMTS(v)$ and $LRDTS(v)$ exist for

$v = 3^k m$, where $k \geq 1$ and $m \in \{1, 4, 5, 7, 11, 13, 17, 23, 25, 35, 37, 41, 43, 47, 53, 55, 57, 61, 65, 67, 91, 123\}$;

$v = 7^k + 2,\ 13^k + 2,\ 25^k + 2,\ 2^{4k} + 2$ and $2^{6k} + 2$ where $k \geq 0$.

And, if there exists an $LRMTS(v)$ (resp. $LRDTS(v)$) then there exist $LRMTS((2 \cdot r^k + 1)v)$ (resp. $LRDTS((2 \cdot r^k + 1)v)$) for $k \geq 0,\ r = 7, 13$ and $v \equiv 0, 3, 9 \bmod 12$.

In this paper, we first give a special structure for $LRMTS(2q + 2)$ and a method to construct $LRMTS(q' + 2)$, where both $q = 6t + 5$ and $q' = 6s + 1$ are prime powers. Then, using computer, the solutions for $t \in T = \{0, 1, 2, 3, 4, 6, 7, 8, 9, 14, 16, 18, 20, 22, 24, 28, 32\}$ and $s \in S = \{35, 38, 46, 47, 48, 51, 56, 60\}$ are found out. Furthermore, using a method introduced by Kang, the corresponding $LRDTS$ are obtained too. Finally, by the tripling construction and product construction for $LRMTS$ and $LRDTS$, and by new results for LR-design, we obtain the existence for $LRMTS(v)$ and $LRDTS(v)$ with orders

$$v = 12(t + 1) \prod_{m_i \geq 0} (2 \cdot 7^{m_i} + 1) \prod_{n_i \geq 0} (2 \cdot 13^{n_i} + 1) \text{ and } t \in T,$$
$$v = 3(2s + 1) \prod_{m_i \geq 0} (2 \cdot 7^{m_i} + 1) \prod_{n_i \geq 0} (2 \cdot 13^{n_i} + 1) \text{ and } s \in S,$$

which provide more infinite families for large sets of resolvable MTS and DTS.

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A new criterion for a Latin square to be group-based

A.D. Keedwell

MSC2000: 05B15, 20N05

We shall describe new sufficient (and necessary) conditions for a Latin square to be group-based. For a Latin square of order n at most n/p easy-to-implement-by-hand tests are required, where p is the smallest prime which divides n. Our method exploits the fact that the middle nucleus of a loop is a group. In particular, for a square of prime order just one test is sufficient. (Compare the $O(n^2)$ tests required to meet Suschkewitch's condition or the quadrangle criterion.)

The rôle of approximate structure in extremal combinatorics

Peter Keevash

MSC2000: 05D05

We discuss the following method for solving problems in extremal combinatorics. In order to show that a given configuration is a unique optimum for an extremal problem, we first prove an approximate structure theorem for all constructions whose value is close to the optimum, and then use this theorem to show that any imperfection in the structure must lead to a suboptimal configuration. We find a new proof of a theorem of Frankl and Füredi (joint work with Dhruv Mubayi) and solve a conjecture of Sós and a conjecture of Frankl (joint work with Benny Sudakov).

Comparing subclasses of well-covered graphs

E.L.C. King

MSC2000: 05C69

A graph G is said to be *well-covered* if every maximal independent set of G is of the same size. It has been shown that characterizing well-covered graphs is a co-NP-complete problem. In an effort to characterize some of these graphs, different subclasses of well-covered graphs have been studied. In this talk, we will discuss the relationships between four of these subclasses: *well-dominated graphs* (those graphs for which every minimal dominating set is minimum), $\alpha = \gamma$ *graphs* (those graphs for which the cardinality of a minimum dominating set is the same as the cardinality of a maximum independent set), *strongly well-covered graphs* (those graphs that remain well-covered with the deletion of any edge), and *stable well-covered graphs* (those graphs - introduced in the speaker's doctoral dissertation - that remain well-covered with the addition of any edge). We illustrate which of these subclasses intersect, which are subsets of one another and which are disjoint from one another.

Reconstruction of permutations from their erroneous patterns

Elena V. Konstantinova

MSC2000: 05C25, 05C85, 05C90

Reconstruction problem arises in graph theory and coding theory as well as in molecular biology if one is interested in reconstructing unknown genetic sequences. We solve the reconstruction problem of permutations and signed permutations on n elements from their erroneous patterns which are distorted by reversal of intervals (with replacing signs in the case of signed permutations). We show that for any $n \geq 2$ an unknown signed permutation is uniquely reconstructible from 3 signed permutations being at the reversal distance at most one from the unknown signed permutation. The reversal distance is defined as the minimal number of reversals of an permutation interval which are needed to transform one permutation into another. Under the same conditions for any $n \geq 3$ an unknown permutation is uniquely reconstructible from 4 permutations. We also investigate the cases when a smaller number of permutations or signed permutations are sufficient to determine an unknown permutation or singed permutation uniquely. A reconstruction algorithm is presented for permutations [1] as well as for signed permutations. The proposed approach is based on an investigation of structural properties of a certain graph constructed for this problem. In particular, it is proved that the considered graph for signed permutations does not contain C_3, C_5 and bipartite subgraphs $K_{2,3}$ and contains C_4. The considered graph for permutations does not contain C_3 and bipartite subgraphs $K_{2,4}$ and contains bipartite subgraphs $K_{3,3}$. It is also shown that in the case of at most two reversal errors it is needed much more different erroneous patterns to reconstruct an unknown permutation or signed permutation.

1. E. V. Konstantinova, "Reconstruction of permutations distorted by single reversal errors", Abstracts of talks of the 2004 IEEE International Symposium on Information Theory ISIT-2004, Chicago, June 27 – July 2, 2004, 451.

Maximum packing for perfect four-triple configurations

Selda Küçükçifçi

(joint work with Güven Yücetürk)

MSC2000: 05B07, 05B40

The graph consisting of the four 3-cycles (triples) $(a, b, h), (b, c, d), (d, e, f)$, and (f, g, h), where a, b, c, d, e, f, g, h are distinct is called a 4-cycle-triple block and the 4-cycle (b, d, f, h) of the 4-cycle-triple block is called an inside 4-cycle. The graph consisting of the four 3-cycles $(a, b, f), (b, c, d), (d, e, f)$, and (f, g, h), where a, b, c, d, e, f, g, h are distinct is called a kite-triple block and the kite $(b, d, f) - h$ (consisting of a 3-cycle with a pendant edge) is called an inside kite. A decomposition of $3kK_n$ into 4-cycle-triple blocks (or into kite triple blocks) is said to be perfect if the inside 4-cycles (or kites) form a k-fold 4-cycle system (or kite system). A perfect maximum packing of $3kK_n$ with 4-cycle-triples (or kite-triples) is a triple (X, T, L), where T is a collection of edge disjoint 4-cycle-triples (or kite-triples) and L is a collection of 3-cycles such that the inside of 4-cycle-triples (or kite-triples) plus the inside of the 3-cycles in L form a maximum packing of kK_n with 4-cycles (or kites).

A complete solution for the problem of constructing perfect $3k$-fold 4-cycle-triple and kite-triple systems was given recently by E.J. Billington, C.C. Lindner, and A. Rosa [1] . In this work, we give a complete solution of the problem of constructing perfect maximum packings of $3kK_n$ with 4-cycle-triples and kite-triples.

Reference.

[1] E.J. Billington, C.C. Lindner, and A. Rosa, *Lambda-fold complete graph decompositions into perfect four-triple configurations*, Australasian Journal of Combinatorics, to appear.

Pseudo 2–factor isomorphic regular bipartite graphs

D. Labbate

(joint work with M. Abreu, B. Jackson and J. Sheehan)

MSC2000: 05C70, 05C75

A graph with a 2–factor is said to be 2*–factor hamiltonian* if all its 2–factors are hamiltonian cycles, and, more generally, 2*–factor isomorphic* if all its 2–factors are isomorphic. Examples of such graphs are K_4, K_5, $K_{3,3}$, the Heawood graph (which are all 2–factor hamiltonian) and the Petersen graph (which is 2–factor isomorphic). Several recent papers have addressed the problem of characterizing families of graphs (particularly regular graphs) which have these properties.

Let G be a graph which contains a 2–factor X. Let t be a $\{0,1\}$–function defined on the 2–factors X of G as follows:

$$t(X) = \begin{cases} 0 & X \text{ has an even number of circuits of length} \equiv 0 \mod 4 \\ 1 & \text{otherwise} \end{cases}$$

Let G be a bipartite graph and suppose that for all 2–factors Y of G, $t(Y) = t$. In this case, we write $t(G) =: t(X)$ and G is said to be *pseudo* 2*–factor isomorphic*.

We prove that:

(1) The class of k–regular bipartite 2–factor isomorphic graphs and pseudo 2–factor isomorphic graphs differs.

(2) The class of pseudo 2–factor isomorphic k–regular bipartite graphs is empty for $k \geq 4$.

Bertrand Postulate, the Prime Number Theorem and product anti-magic graphs

A. Lev

(joint work with G. Kaplan and Y. Roditty)

MSC2000: 05C78

Let the edges of the finite simple graph $G = (V, E)$, $|V| = n$, $|E| = m$ be labeled by the integers $1, 2, \dots, m$. Denote by $w(u)$ the product of all the labels of edges incident with a vertex u. The graph G is called *product anti-magic* if it is possible that the above labeling results in all values $w(u)$ being distinct.

An old conjecture of Ringel states that every connected graph, but K_2, is product anti-magic. In this paper we prove this conjecture for dense graphs, complete bipartite graphs and some other families of graphs.

Reconstruction of graphs from metric balls of their vertices

Vladimir I. Levenshtein

MSC2000: 05C12, 05C35, 05C60

A new problem of reconstruction of a simple connected graph $G = (V, E)$ from metric balls $B_r(x, G)$ of a given radius r ($r \geq 2$) centred at all its vertices $x \in V$ is considered. We say that a graph $G = (V, E)$ of a family F is *reconstructible* from metric balls of a given radius r ($r \geq 2$) if any two graphs $G = (V, E)$ and $G' = (V, E')$ of this family (with the same vertex set V), for which $B_r(x, G) = B_r(x, G')$ for all $x \in V$, coincide (i.e., $E = E'$). This reconstruction problem introduced in [1] is motivated by applications in chemistry for the structure elucidation of unknown compounds and has quiet different nature compared with the classical Ulam's problem. In [1] it is proved that any graph $G = (V, E)$, which has at least 3 non–terminal vertices and whose girth $g(G)$ is at least 7, is reconstructible from metric balls of radius 2 of all its vertices and it is shown that these sufficient conditions are necessary in a sense. The problem of reconstruction of an unknown graph from metric balls of radius larger than 2 requires stronger restrictions. Let $F(t)$ be the family of simple connected graphs $G = (V, E)$ without terminal vertices for which $g(G) \geq t$. For a fixed $r \geq 2$, denote by $t(r)$ the minimum t such that any graph $G \in F(t)$ is reconstructible from metric balls of radius r of all its vertices. Cyclic graphs on $2r + 2$ vertices are not reconstructible and this implies $t(r) \geq 2r + 3$. The author conjectures that $t(r) = 2r + 3$ for all $r \geq 2$. However, so far the author has a proof only of the following upper bound: $t(r) \leq 2r + 2\lceil \frac{r-1}{4} \rceil + 1$. This implies that the conjecture above is valid for $r = 2, 3, 4, 5$.

1. V.I. Levenshtein, E. Konstantinova, E. Konstantinov, S. Molodtsov, Reconstruction of a graph from 2-vicinities of its vertices, accepted for publication in Discrete Applied Mathematics.

Polynomial variants of the densest/heaviest k-subgraph problem

Maria Liazi

(joint work with Vassilis Zissimopoulos and Ioannis Milis)

MSC2000: 05C85, 68Q25, 68W40

In the *Densest k-subgraph* (DkS) problem we are given a graph $G = (V, E)$, with $|V| = n$, and an integer k, $3 \leq k \leq n$, and we ask for a set of k vertices such that the number of edges in the subgraph of G induced by this set is maximized. The *Heaviest k-subgraph* (HkS) problem is the weighted version of the DkS: the edges of the given graph have non negative weights and the goal is to find the k-vertex induced subgraph with maximum total edge weight. Both problems are NP-hard as generalizations of the well known *Clique* problem.

Although several approximation algorithms have been proposed for the general case of both problems, no one of them achieves a constant approximation ratio nor we have a complementary negative inapproximability result.

Concerning special cases of the DkS problem it is known that it remains NP-hard for a number of special graph classes including bipartite graphs (even of maximal degree three), regular graphs, comparability graphs, chordal graphs and planar graphs. DkS is trivial on trees, while polynomial time algorithms are known for graphs of maximal degree two, cographs, split graphs and k-trees.

On the other hand it is known that the HkS problem is polynomial on trees under the restriction that the solution we are looking for is connected (i.e. a single subtree of the input tree). However, in general an optimal solution to the HkS problem on a tree could be disconnected.

In this paper we focus on the direction of further identifying the frontier between polynomial and NP-hard cases of the DkS and HkS problems with respect to the class of the input graph. First, we propose two $O(nk^2)$ time algorithms yielding optimal (either connected or disconnected) solutions for the HkS problem on trees and on graphs of maximal degree two. We also propose an $O(nk^4)$ dynamic programming algorithm for the connected DkS problem on a subclass of interval graphs.

Cycles in a tournament with pairwise zero, one or two given common vertices

Nicolas Lichiardopol

MSC2000: 05C20

G. Chen, R.J. Gould and H. Li proved in [1] that every k-connected tournament with at least $8k$ vertices admits k vertex-disjoint cycles spanning the vertex set, which answered to a question posed by B. Bollobas (see [2])

In this talk, we prove, as consequence of a more general result, that every k-connected tournament of diameter of least 4 admits k vertex-disjoint cycles spanning the vertex set.

Then, for a connected tournament T of diameter at most 3, we determine a relation between the maximum number of vertex-disjoint cycles and the maximum number of vertex-disjoint cycles spanning the vertex set of T. By using also a Lemma of [1], we prove that a k-connected tournament of order at least $5k-3$, of diameter 2 (resp. 3) admits k (resp. $k-1$) vertex-disjoint cycles spanning the vertex set.

At last, we give results on cycles with pairwise one or two given common vertices.

Some open problems will be raised.

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Combinatorial families enumerated by quasi-polynomials

P. Lisoněk

MSC2000: 05A15

We say that the sequence (a_n) is *quasi-polynomial* in n if there exist polynomials $P_0, ..., P_{s-1}$ such that $a_n = P_i(n)$ where $i \equiv n \pmod{s}$. We present several families of combinatorial structures with the following properties: Each family of structures depends on two or more parameters, and the number of isomorphism types of structures is quasi-polynomial in one of the parameters whenever the values of the remaining parameters are fixed to arbitrary constants. For each family we are able to translate the problem of counting isomorphism types of structures to the problem of counting integer points in a union of parameterized rational polytopes. The quasi-polynomiality of the counting sequence then follows from Ehrhart's result about the number of integer points in the sequence of integral dilates of a given rational polytope. The families of structures to which this approach is applicable include combinatorial designs, linear and non-linear codes, and dissections of regular polygons.

Eccentricity sequences and eccentricity sets in digraphs

N. López

(joint work with J. Gimbert)

MSC2000: 05C12, 05C20

The eccentricity $e(v)$ of a vertex v in a strongly connected digraph G is the maximum distance from v. The eccentricity sequence of a digraph is the list of eccentricities of its vertices given in nondecreasing order. A sequence of positive integers is a *digraphical eccentric sequence* if it is the eccentricity sequence of some digraph. A set of positive integers S is a *digraphical eccentric set* if there is a digraph G such that $S = \{e(v),\ v \in V(G)\}$. In this talk, we present some necessary and sufficient conditions for a sequence S to be a digraphical eccentric sequence. In some particular cases, where either the minimum or the maximum value of S is fixed, a characterization is derived. We also characterize digraphical eccentric sets.

On the metric dimension of graph products

María Luz Puertas

(joint work with José Cáceres, Carmen Hernando, Mercè Mora, Ignacio M. Pelayo, Carlos Seara and David R. Wood)

MSC2000: 05C12, 05C38

A vertex x of a graph G is said to resolve two vertices u and v of G if $d(x,u) \neq d(x,v)$. An ordered vertex set S of a graph G is a *resolving set* of G if every two distinct vertices of G are resolved by some vertex of S. The concept of (minimum) resolving set of a graph has proved to be useful and/or related to a variety of fields such as Chemistry [3], Robotic Navigation [2] and Combinatorial Search and Optimization [4].

This work is devoted to evaluating the so-called *metric dimension* [1, 5] of finite connected graphs, i.e., the minimum cardinality of a resolving set. Firstly we find a non-trivial universal resolving set, and then we focus our attention on cartesian products of graphs. We show some results about upper and lower bounds of the metric dimension of the product $G \times H$ of two graphs and we study in detail particular cases, such as products of complete graphs, cycles or paths. In these cases we provide exact values of the metric dimension and we also describe minimum resolving sets of cartesian product.

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On the strong circular 5-flow conjecture

E. Máčajová

(joint work with A. Raspaud)

MSC2000: 05C15, 90B10

The strong circular 5-flow conjecture of B. Mohar claims that each snark, with the single exception of the Petersen graph, has circular flow number smaller than 5. We disprove this conjecture by constructing an infinite family of cyclically 4-edge connected snarks with circular flow number exactly 5.

"Almost stable" matchings in the Roommates problem

D.F. Manlove

(joint work with D.J. Abraham and P. Biró)

MSC2000: 05C70, 68Q17, 68W25, 91B68

The Stable Roommates problem (SR) is a classical combinatorial problem. An instance of SR involves $2n$ agents, each of whom ranks all others in strict order of preference. A *matching* M is a set of n disjoint pairs of agents. A *blocking pair* of M is a pair of agents, each of whom prefers the other to their partner in M. A matching is *stable* if it admits no blocking pair. It is known that an SR instance need not admit a stable matching. This motivates the problem of finding a matching that is "as stable as possible", i.e. admits the fewest number of blocking pairs. We show that, given an SR instance I, the problem of finding a matching with the fewest number of blocking pairs is NP-hard and very difficult to approximate. On the other hand, given a constant K, we show that the problem of finding a matching with at most K blocking pairs, or reporting that no such matching exists, is solvable in polynomial time.

On the connectivity of a product of graphs

X. Marcote

(joint work with C. Balbuena, M. Cera, A. Diánez, and P. García-Vázquez)

MSC2000: 05C35, 05C40

The product graph $G_m * G_p$ of two given graphs G_m and G_p was defined by J.C. Bermond, C. Delorme, and G. Farhi [J. Combin. Theory, Series B 36 (1984) 32-48] in the context of the so-called (Δ, D)-*problem*, and can be seen as an interesting model in the design of large reliable interconnection networks. This work deals with product graphs $G_m * G_p$ for which we provide bounds for two connectivity parameters (λ and λ', edge-connectivity and restricted edge-connectivity, respectively) and present some sufficient conditions to guarantee optimal values of these parameters. The obtained results are compared with other previous related ones for permutation graphs and cartesian product graphs. A similar approach can be carried out for the vertex-connectivity of product graphs.

Special sets of the Hermitian surface and Segre invariants

G. Marino

(joint work with A. Cossidente and O.H. King)

MSC2000: 51E21, 51E14

A *special set* $\mathcal{S}$ of the Hermitian surface $\mathcal{H}(3, q^2)$ of $\mathrm{PG}(3, q^2)$ is a set of q^2+1 points such that any three of them generate a secant plane to $\mathcal{H}(3, q^2)$. A characterization of certain elliptic quadrics $\mathcal{Q}^-(3, q)$ embedded in $\mathcal{H}(3, q^2)$, q odd, as special sets, in terms of Segre invariants, is given.

Unbalanced $K_{p,q}$ factorisations of complete bipartite graphs

N. Martin

MSC2000: 05C70

A $K_{p,q}$ factor of $K_{m,n}$ is a spanning subgraph all of whose components are copies of $K_{p,q}$. In such a factor we will find two types of $K_{p,q}$ one where the p-set of a $K_{p,q}$ is in the m-set of $K_{m,n}$ and the other in the n-set of $K_{m,n}$. We call the ratio of these respective types the *balance ratio* of the factor and label it with integers $x : y$ chosen so that $\gcd(x, y) = 1$ [so the actual numbers of oriented components are respectively dx and dy for some integer d]. A $K_{p,q}$ factorization of $K_{m,n}$ is a decomposition of $K_{m,n}$ into edge-disjoint $K_{p,q}$ factors. All factors in a factorization must have the same balance ratio.

It is conjectured that $K_{p,q}$ factorizations of $K_{m,n}$ always exist when a small number of necessary simple arithmetical conditions exist. In attacking this conjecture it is sufficient to deal with the case where $\gcd(p, q) = 1$ and, for a given, balance ratio $x : y$ to exhibit a $K_{p,q}$ factorizations of $K_{m,n}$ where $m = (qx + py)d,\ n = (px + qy)d$ and d is the denominator of the fraction $\frac{(q-p)xy}{pq(x+y)}$ expressed in its lowest form [we assume that $q > p$].

The conjecture has been proved for all pairs (p, q) when $x = y = 1$ [the balanced case] and for all pairs (x, y) when $(p, q) = (1, 2),\ (1, 3),\ (2, 3)$ as well as for several infinite families of other general values.

In this paper we first recalculate a condition from the first of these infinite families, arising from a regular tiling of the plane, and show that the conjecture is true in all cases where $\gcd(p, x) = \gcd(q, y) = \gcd(q - p, x + y) = 1$. We then improve this with a new construction to show that the conjecture is true whenever just $\gcd(q - p, x + y) = 1$. An immediate consequence is that the conjecture is true for all $K_{p,p+1}$-factorizations of complete bipartite graphs.

On optimal non-projective ternary linear codes

T. Maruta

(joint work with M. Takenaka and K. Okamoto)

MSC2000: 94B05, 94B65, 51E20

We denote by $n_q(k, d)$ the minimum length n for which an $[n, k, d]_q$ code exists. For ternary linear codes, $n_3(k, d)$ is known for $k \leq 5$ for all d. We try to find optimal ternary linear codes of dimension 6 with the minimum distance $d > 243$, which are neccessarily non-projective. The exact value of $n_3(6, d)$ is determined for $d \in \{268 - 270, 280 - 282, 304 - 306, 313 - 315, 347, 348\}$.

On spanning trees with degree restrictions

H. Matsumura

(joint work with H. Enomoto and H. Matsuda)

MSC2000: 05C05

A k-tree is a spanning tree with maximum degree at most k. A degree sum condition for a graph to have a k-tree was given by Win. In this talk, we consider the following problems:

Let $k \geq 2$ be an integer, G be a connected graph and $S \subset V(G)$. Find the sufficient condition to contain a k-tree T satisfying

(1) $\deg_T(x) = 1$ for any $x \in S$, or

(2) $\deg_T(x) < k$ for any $x \in S$.

We also propose a conjecture on more general case.

Doubly transitivity on 2-factors

G. Mazzuoccolo

MSC2000: 05C25, 05C15, 05C70

A 2-factor in a graph Γ is a 2-regular spanning subgraph and a 2-factorization of Γ is a partition of the edge-set of Γ into edge-disjoint 2-factors.

Various assumptions on the automorphism group have been considered when Γ is the complete graph K_v, but they generally deal with the action of the group on the vertex-set. In this talk we consider 2-factorizations of K_v admitting an automorphism group G acting doubly transitively on the set of factors. In the Hamiltonian case the only possibility is the unique factorization of K_5, while in the non-Hamiltonian one we give some infinite classes of examples and one sporadic construction. Finally we also give some necessary conditions for the existence of such factorizations.

The Path Partition Conjecture

K.L. McAvaney

(joint work with R.E.L. Aldred)

MSC2000: 05C38

Let $\tau(G)$ denote the number of vertices in a longest path in a graph G. Given a pair of positive integers a and b, we say G is (a, b)-partitionable if there is a partition $\{A, B\}$ of its vertices so that $\tau(G[A]) \leq a$ and $\tau(G[B]) \leq b$. If G is (a, b)-partitionable for all a and b with $a + b = \tau(G)$, we say G is path partitionable. Immediate examples are any hamiltonian or bipartite graph. The Path Partition Conjecture (Laborde et al. 1983) asserts that all graphs are path partitionable. We briefly review past work on this elusive conjecture and outline some recent results.

Random planar graphs and related structures

Colin McDiarmid

MSC2000: 05C80

We consider the behaviour of the random planar graph, drawn uniformly at random from the set of all simple planar graphs on vertices $1, \dots, n$, and the behaviour of related random structures. We discuss recent work of Gerke, Steger, Welsh, Weissl and the speaker, and of Giminez and Noy, and give some extensions. For example, we see that if we replace the plane by any given surface then we obtain exactly the same growth constants.

Short cycles in random regular graphs

Brendan D. McKay

(joint work with Nicholas C. Wormald and Beata Wysocka)

MSC2000: 05C80

Consider random regular graphs of order n and degree $d = d(n) \geq 3$. Let $g = g(n) \geq 3$ satisfy $(d-1)^{2g-1} = o(n)$. Then the numbers of cycles of lengths up to g have a distribution similar to that of independent Poisson variables. In particular, we find the asymptotic probability that there are no cycles with sizes in a given set, including the probability that the girth is greater than g. A corresponding result is given for random regular bipartite graphs.

We also describe recent extensions by Gao and Wormald.

On the number of tilings of rectangles with T-tetraminoes

C. Merino

MSC2000: 05A16, 82B20

The classical combinatorial problem of counting domino tilings of an $m \times n$ rectangle was solved by P.W. Kasteleyn and also by H.N.V. Temperley and M.E. Fisher in 1961.

We shall consider the same problem but for T-tetraminoes, that is, pieces formed by 4 unit squares in the shape of a T. The number of such tilings has been proved to be an evaluation of the Tutte polynomial of an associated rectangular lattice. Here we present some results about the number of T-tetramino tilings for $4 \times n$, $8 \times n$ and $12 \times n$ rectangles.

On the Frobenius problem of three numbers: Part I

Alícia Miralles

(joint work with F. Aguiló and M. Zaragozá)

MSC2000: 05C20, 10A50, 11D04.

Given a set $A = \{a_1, ..., a_k\} \subset \mathbb{N}$, with $\gcd(a_1, ..., a_k) = 1$, let us define

$$R(A) = \{\sum_{i=1}^{k} \lambda_i a_i | \ \lambda_1, ..., \lambda_k \in \mathbb{N}\},$$

and $\overline{R}(A) = \mathbb{N} \setminus R(A)$. It can be easily seen that $|\overline{R}(A)| < \infty$. The *Frobenius problem* related to A, FP(A), consists on the study of the set $\overline{R}(A)$. The *solution* of FP(A) is the explicit description of $\overline{R}(A)$, however this is a difficult task. Usually *partial solutions* are given, like the cardinal $|\overline{R}(A)|$ and/or the *Frobenius number* $f(A) = \max \overline{R}(A)$.

In this work we give a method to find the solution of FP(A), with $k = 3$. Using the notation $A = \{a, b, N\}$ with $N = \max A$, we use the *Double-loop* digraph $G = G(N; a, b)$ as a tool to solve FP(A). Each digraph G has linked a metrical diagram known as *Minimum Distance Diagram* (MDD) which is an L-shaped tile. This MDD gives metrical information of the equivalent classes modulus N. The solution of FP(A) can be explicitly given from another kind of diagram which we call the *Minimum Distance Diagram of Elements* (MDDE,) which gives metrical information of $\overline{R}(A)$.

We give a characterization of the MDD which are MDDE also, and therefore we are able to solve the FP(A). This method allows us to solve Frobenius problems of symbolical nature, which can not be solved by the known numerical algorithms. To give an example, we propose an infinite sequence of sets $A_n = \{a_n, b_n, N_n\}$ and its related sequence of Frobenius solutions $\overline{R}(A_n)$.

On the zero-divisor graph of a ring

A. Mohammadian

(joint work with S. Akbari)

MSC2000: 05C20, 05C69, 16P10

The study of algebraic structures, using the properties of graphs, has become an exciting research topic in the last twenty years, leading to many fascinating results and questions. In this talk we study the zero-divisor graph of a ring and investigate the interplay between the ring-theoretic properties of a ring and the graph-theoretic properties of its zero-divisor graph.

Suppose that R is an arbitrary ring. The zero-divisor graph of the ring R, denoted by $\Gamma(R)$, is a directed graph whose vertices are all non-zero zero-divisors of R, in which for any two distinct vertices x and y, $x \to y$ is an edge if and only if $xy = 0$. Also for a ring R, we define a simple undirected graph $\overline{\Gamma}(R)$ whose vertices are all non-zero zero-divisors of R, in which two distinct vertices x and y are adjacent if and only if either $xy = 0$ or $yx = 0$. In this talk we discuss on some graph-theoretic properties of $\Gamma(R)$ and $\overline{\Gamma}(R)$ and determine some graph-theoretic parameters of these graphs. Recently S. P. Redmond has proved that for any finite ring R, the graph $\Gamma(R)$ has an even number of edges. We give a simple proof for this result. We will express some results about $\Gamma(R)$ and $\overline{\Gamma}(R)$ appeared in [1] and [2].

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Domination number of some 3−regular graphs

DoostAli Mojdeh

(joint work with H. Abdollahzadeh Ahangar, and A. Ahmadi Haji)

MSC2000: 05C69

The subset $S \subseteq V$ of the vertices in a graph $G = (V, E)$ is called a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S. The domination number, $\gamma(G)$ of G is the minimum cardinality among the dominating sets of G. A dominating set S is also called an independent dominating set of G if every two vertices of S are not adjacent. The minimum cardinality of an independent dominating set of G is the independent domination number $i(G)$. A dominating set S is called connected dominating set if $\langle S \rangle$ is connected and the connected domination number, $\gamma_c(G)$ of G is the minimum cardinality among the connected dominating sets of G. A subset T of a minimum dominating set S is a forcing subset for S if S is the unique minimum dominating set containing T. The forcing domination number $f(G, \gamma)$ of G, is the minimum cardinality among the minimum dominating sets of G. The dominating set of regular graphs have been studied yet, but there exist the bounds for the domination number of them, (See [1,2,3,4,5] for furthermore).

In this note we study the $\gamma(G)$, $i(G)$, $\gamma_c(G)$ and $f(G, \gamma)$ for some 3−regular graph and we obtain a sharp value.

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New bounds on some Turán numbers for infinitely many n

B. Montágh

MSC2000: 05C35

A construction of $K_{3,l}$-free graphs of order n and size $(r^{2/3}/2 + o(1))n^{5/3}$ will be presented, with $r = \lfloor\sqrt{(l-1)/2}\rfloor$. The main term matches the construction of Alon, Rónyai and Szabó, the best previously known. If $r \geq 3$ (that is, $l \geq 19$), then, for infinitely many n, our error term is larger. In these cases we obtain a $K_{3,l}$-free graph of larger size than any previously known $K_{3,l}$-free graph of the same order.

Codes, Designs and Graphs from Finite Simple Groups

J. Moori

(joint work with J.D. Key and B. Rodrigues)

MSC2000: 05B05, 20D08, 94D08

Error-correcting codes that have large automorphism groups whose properties are extensively studied can be useful in applications as the group can help in determining the code's properties, and can be useful in decoding algorithms by finding PD-sets.

We consider primitive representations of a simple group G. For each group, using Magma, we construct designs and graphs that have the group acting primitively on points as automorphism group, and, for a selection of small primes, codes over that prime field derived from the designs or graphs that also have the group acting as automorphism group. For each code, the code automorphism group at least contains the associated group G. We have considered various groups, for example J_1, J_2, M^cL and $PSp_{2m}(q)$, where q is a power of an odd prime, and $m \geq 2$. Most of these results have appeared in a series of papers written with J D Key and B Rodrigues.

Maximal increasing paths in edge-ordered trees

Kieka Mynhardt

(joint work with Ernie Cockayne)

MSC2000: 05C38, 05C78

An edge ordering of a simple graph $G = (V, E)$ is an injection $f : E \to \mathbb{N}$. Denote the set of all edge orderings of G by $\mathcal{F}(G)$. A (simple) path λ in G for which $f \in \mathcal{F}(G)$ increases along its edge sequence is called an f-ascent of G. An f-ascent is called maximal if it is not contained in a longer f-ascent of G. Let $h(f)$ denote the length of a shortest maximal f-ascent and define $\varepsilon(G) = \max_{f \in \mathcal{F}(G)}\{h(f)\}$, that is, $\varepsilon(G)$ is the smallest integer k such that every edge ordering of G has a maximal ascent of length at most k. Obviously $\varepsilon(G) = 1$ if and only if $\Delta(G) = 1$, and it can be shown that $\varepsilon(G) = 2$ if and only if G has a vertex adjacent to two leaves, or to two adjacent vertices of degree two.

We determine a formula for ε for trees in which no two branch vertices are adjacent, show that this formula does not hold otherwise and characterise trees with $\varepsilon = 3$.

Broken circuits and NBC complexes of convex geometries

M. Nakamura

(joint work with K. Kashiwabara)

MSC2000: 05A99, 05B35, 06C10

A convex geometry is a closure system whose closure operator satisfies the anti-exchange property, while a closure system is a set of flats of a matroid if and only if the associated closure operator meets the exchange property.

For a matroid with a linear order on the underlying set, a broken circuit is a set of the form $C \setminus e_C$ where C is a circuit and e_C is the minimum element in C. An NBC complex is the collection of those sets which containing no broken circuits. An NBC complex plays a crucial role in the Whitney-Rota's formula for the characteristic polynomials of matroids, the NBC basis theorem of the Orlik-Solomon algebra, and so on.

We introduce a notion of *a broken circuit* of a convex geometry as a set obtained from a circuit of the convex geometry by deleting its root. As is the same with that of a matroid, an NBC complex of a convex geometry is defined as the collection of those sets containing no broken circuits. (Note that for the definition of a broken circuit of a convex geometry, we need not to assume a linear order on the underlying set.) Our definition can be justified by the fact that we can establish the following results of convex geometries analogous to those of matroids.

(1) Whitney-Rota's formula holds for the characteristic polynomial $p(K;\lambda)$ of a convex geometry K as

$$p(K;\lambda) = \sum_{X \in NBC(K)} (-1)^{|X|} \lambda^{|E|-|X|}.$$

(2) We have a decomposition of the NBC complex $NBC(K)$ of a convex geometry K, with respect to a coloop x, as

$$NBC(K) = NBC(K \setminus x) \uplus (NBC(K/x) * x).$$

This is a complete analogue of Brylawski's decomposition of NBC complexes of matroids.

(3) We can define an Orlik-Solomon type algebra $A(K)$ for a convex geometry K so that we have a short exact split sequence among them below.

$$0 \;\longrightarrow\; A(K \setminus x) \;\xrightarrow{i_x}\; A(K) \;\xrightarrow{p_x}\; A(K/x) \;\longrightarrow\; 0.$$

Orthogonality graphs from quantum computing

M.W. Newman

(joint work with C.D. Godsil)

MSC2000: 05C15

We deal with a question in graph colouring motivated by an application from quantum computation.

The graph Ω_n has vertex set all ± 1-vectors of length n, where two vertices are adjacent if they are orthogonal as vectors: we wish to know when $\chi(\Omega_{2^k}) = 2^k$. We show that this is the case precisely when $k \leq 3$. Our methods are algebraic, and in particular the Delsarte-Hoffman bound on independent sets plays a crucial role. The technique we use also has a wider application.

We will briefly describe the motivating problem, but focus mainly on the graph theory; no prior knowledge of quantum computing is necessary.

The strong metric dimension of graphs

Ortrud R. Oellermann

(joint work with Joel Peters-Fransen)

MSC2000: 05C12, 05C85

Let G be a connected (di)graph. A vertex w strongly resolves a pair u, v of vertices of G if there exists some shortest $u-w$ path containing v or some shortest $v-w$ path containing u. A set W of vertices is a strong resolving set for G if every pair of vertices of G is strongly resolved by some vertex of W. The smallest strong resolving set for G is called a strong basis for G and its cardinality the strong dimension of G. (Sebö and Tannier introduced these concepts when studying extensions of isometries between metric spaces.) It will be shown in this talk that
(i) the problem of finding the strong dimension of a connected graph can be transformed to the problem of finding the vertex covering number of a graph and
(ii) that the problem of finding this invariant is NP-hard.

The intricacy of avoiding arrays

L–D. Öhman

MSC2000: 05B15, 05C15

A *Latin square* L is a square $n \times n$ array on the symbols $1, 2, \dots, n$ where each symbol is used exactly once in each row and column. A square array A is *avoidable* if there exists some Latin square L of the same order as A whose entries never coincide with the corresponding entries in A. Obviously, there are unavoidable arrays. We ask the question of the *intricacy* of avoiding general arrays, with one or more entries in each cell. The intricacy of this problem is the natural number $I(m, n)$ that answers the question: "What is the minimum number of avoidable arrays that any $n \times n$ array A with at most m entries in each cell can be partitioned into?" It is shown that for any $n \geq 2$ it holds that $I(1, n) = 2$, and $I(n-1, n) = n$. Further, it is shown that $\lceil \frac{n}{n-m} \rceil \leq I(m, n) \leq \lceil \frac{n}{n-m} \rceil + 3$. It is conjectured that $I(m, n) = \lceil \frac{n}{n-m} \rceil$.

On the domatic number of the 2-section graph of the order-interval hypergraph of a finite poset

S. Ouatiki

(joint work with I. Bouchemakh)

MSC2000: 05C35, 05C65, 05C69, 06A07, 68R10, 90C27

Given a finite poset P, let $\mathcal{H}(P)$ be the hypergraph whose vertices are the points of P and whose edges are the maximal intervals in P. The purpose of this paper is to study the domatic number $d(G(P))$ of the 2-section graph $G(P)$ of the hypergraph $\mathcal{H}(P)$. For the subset $P_{l,u}$ of P induced by consecutive levels $\cup_{i=l}^{u} N_i$ of P, we give exact values of $d(G(P_{l,u}))$ when P is the chain product $C_{n_1} \times C_{n_2}$. According to the values of l, u, n_1, n_2, the maximal domatic partition is exhibited. Moreover, we give some exact values or lower bounds for $d(G(P * Q))$, when $*$ is either the direct sum or the linear sum. Finally we show that the domatic number and the total domatic number problems in this class of graphs are NP-complete.

Graph equivalence from equivalent quantum states

Matthew G. Parker

(joint work with Lars Eirik Danielsen and Constanza Riera)

MSC2000: 05C69, 05C99, 05B20, 06E30

Pure quantum states are equivalent if one state can be obtained from the other by the action of a *local unitary transform* on the state. For quantum bit (*qubit*) systems, such a transform can be written as the tensor product of 2×2 unitary matrices over the complex numbers, where the quantum state is represented as a complex vector. Recent research has identified that so-called *cluster states*, which are pure multipartite quantum states, are favourable candidates from which to build quantum computers. These states have a convenient correspondence to simple graphs. We identify the equivalence of quantum states with certain graph equivalences. Glynn has shown that the action of *local complementation* on a graph leaves the corresponding cluster state invariant, where local complementation was defined by Bouchet in the context of *isotropic systems*. We identify local complementation with the action of local unitary transforms on the vector representing the quantum state, where the transform comprises tensor products of members of the *Local Clifford Group*: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, and $N = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$, where $i^2 = -1$. The action of *pivot* on a graph also leaves the cluster state invariant, where pivot corresponds to tensor products of I and H. Recent work by Arratia, Bollobas and Sorkin, Aigner and van der Holst, and Monaghan and Sarmiento has defined *Interlace Polynomials* for a graph, and these polynomials summarise the spectra of cluster states with respect to tensor products of I, H, and N. Graphs corresponding to cluster states may also be interpreted as additive codes over GF(4) and/or GF(2). There is also a link to *boolean functions*: Let Γ be the adjacency matrix of the simple graph corresponding to a cluster state. Then the n-qubit cluster state can be identified with a quadratic boolean function, $p(x) = \sum_{i=0}^{n-2}\sum_{j=i+1}^{n-1}\Gamma_{ij}x_ix_j$, and the local unitary transformation then corresponds to a generalised measure of cryptographic strength for the boolean function. The transform approach to graph symmetry can be generalised to *hypergraphs* (i.e. to boolean functions of degree > 2). We demonstrate how pivot and local complementation can be generalised to hypergraphs, and also identify hypergraph equivalences which exploit local unitary transforms other than I, H, and N. Hypergraphs correspond to *hyper-cluster states* which do not appear to have received much (if any) attention in the physics community.

The computational complexity of the parallel knock-out problem

D. Paulusma

(joint work with H.J. Broersma, M. Johnson and I.A. Stewart)

MSC2000: 03D15, 05C85

Consider the following *parallel knock-out scheme* for graphs: Every vertex v of an undirected graph selects exactly one of its neighbors. Then all the selected vertices are eliminated simultaneously, and the procedure is repeated with the subgraph induced by the remaining vertices. The procedure terminates as soon as

1. there are no vertices left, or

2. one of the remaining vertices has degree zero in the resulting subgraph.

For all fixed positive integers k we determine the computational complexity of the problem whether a given graph admits a parallel knock-out scheme in which all vertices are eliminated in at most k rounds. We will do this for several graph classes (general, bipartite, bounded tree-width).

Extremal Ramsey graphs

D.B. Penman

(joint work with E. Maistrelli)

MSC2000: 05C55

Let $R(k,k)$ be the smallest number such that any (simple) graph on $R(k,k)$ vertices has either a complete subgraph of order k or an induced null subgraph of order k. An extremal Ramsey graph ERG(k) is a graph on $R(k,k)-1$ vertices which has neither a complete graph of order k nor an induced null subgraph of order k. Until recently, the sum total of our knowledge of these graphs has been perilously close to the statements that the unique ERG(3) is P_5 and that the unique ERG(4) is P_{17}: here, for a prime power q congruent to 1 modulo 4, P_q is the Paley graph on vertex set $\mathbf{F}_q$, two vertices being adjacent if and only if their difference is a non-zero square in the field. The other rough idea floating around has been that the extremal graphs should be not unlike random graphs $G(n,1/2)$, though Thomason has shown that this idea needs to be handled with some degree of caution.

In this talk, I shall describe some preliminary investigations, jointly with my student Eleni Maistrelli, of these graphs.

Pancyclic PBD block-intersection graphs

David A. Pike

(joint work Graham A. Case)

MSC2000: 05C38, 05B05

A pairwise balanced design PBD$(v, \mathcal{K}, \lambda)$ consists of a set V of cardinality v, a set $\mathcal{K}$ of positive integers, and a set B of subsets of V with the properties that $|b| \in \mathcal{K}$ for each $b \in B$, and each pair of elements from V occurs in exactly λ of the subsets in B. The elements of B are known as the blocks of the design.

Given a combinatorial design D with block set B, its block-intersection graph G_D is the graph having vertex set B such that two vertices b_1 and b_2 are adjacent if and only if b_1 and b_2 have non-empty intersection.

Hare showed in 1995 that if D is a PBD$(v, \mathcal{K}, 1)$ with $\min\{\mathcal{K}\} \geqslant 3$, then G_D is edge-pancyclic (i.e. each edge of G_D is contained in a cycle of each length $\ell = 3, 4, \ldots, |V(G_D)|$). In this presentation we consider block-intersection graphs of pairwise balanced designs PBD$(v, \mathcal{K}, \lambda)$ for which $\lambda \geqslant 2$.

Fragmentability of bounded degree graphs

Oleg Pikhurko

(joint work with Penny Haxell)

MSC2000: 05C35

Given a real $\alpha > 0$ and a positive integer f, we say that a graph G is (α, f)-*fragmentable* if there is a set $A \subset V(G)$ such that $|A| \leq \alpha\, v(G)$ and every component of $G - A$ has at most f vertices.

For an integer d, let α_d be the infimum of those α for which there is an f such that every graph with maximum degree at most d is (α, f)-fragmentable. Answering a question of Edwards and Farr posed at BCC18 we will show that

$$\sup\{\alpha_d : d \in \mathbb{N}\} = 1.$$

In fact, we proved the more precise estimate $\alpha_d = 1 - \Theta(d^{-1})$. Also, for a typical random d-regular graph, the appropriately defined infimum of α is $1 - (2 + o(1))\frac{\ln d}{d}$.

Domination in a graph with a 2-factor

Michael D. Plummer

(joint work with K. Kawarabayashi and A. Saito)

MSC2000: 05C69, 05C70

The cardinality of any smallest dominating set in a graph G is called the *domination number* of G and denoted by $\gamma(G)$. In 1996, Reed proved that every graph G of minimum degree at least three satisfies $\gamma(G) \leq (3/8)|V(G)|$ and conjectured that if G is a connected cubic graph, then $\gamma(G) \leq \lceil |V(G)|/3 \rceil$.

Theorem 1. Let G be a connected graph with a 2-factor F and let k be any positive integer. If F has at least two components and the order of each component is at least $3k$, then

$$\gamma(G) \leq \left(\frac{3k+2}{9k+3}\right)|V(G)|.$$

Theorem 2. Let k be any positive integer. Then every 2-edge-connected cubic graph of girth at least $3k$ satisfies

$$\gamma(G) \leq \left(\frac{3k+2}{9k+3}\right)|V(G)|.$$

Note that for girth at least nine, one then has $\gamma(G) \leq (11/30)|V(G)|$, which improves Reed's $(3/8)|V(G)|$ bound.

Quantum error correction codes invariant under symmetries of the square

H. Pollatsek

(joint work with M.B. Ruskai)

MSC2000: 81P68

Quantum error correction is now well-developed in the case of "stabilizer codes," which arise as subspaces of C^{2^n} stabilized by Abelian subgroups of the Pauli group (generated by bit-flips and phase errors). These codes, also known as *additive codes*, can be regarded as generalizations of classical codes.

In previous work, we studied a natural generalization of stabilizer codes to non-additive codes associated with the action of the symmetric group. ("Permutationally invariant codes for quantum error correction," *Linear Algebra and its Applications*, **392** (2004), pp.255-288.)

Now we consider the geometry of the physical arrangement of the qubits comprising the quantum system. For qubits arranged in a square, we study codes invariant under the dihedral group of order 8. For the cases of 4, 5 and 8 qubits, we find infinitely many non-additive codes detecting single errors and able to correct families of single errors.

The talk will not presuppose familiarity with quantum computation; arguments will use algebra and combinatorics.

Some $\mathbb{Z}_{n+2}$ terraces from $\mathbb{Z}_n$ power-sequences, n being an odd prime power

D.A. Preece

(joint work with Ian Anderson)

MSC2000: 11A07, 05B30

A terrace for $\mathbb{Z}_m$ is an arrangement $(a_1, a_2, \ldots, a_m)$ of the m elements of $\mathbb{Z}_m$ such that the sets of differences $a_{i+1} - a_i$ and $a_i - a_{i+1}$ $(i = 1, 2, \ldots, m-1)$ between them contain each element of $\mathbb{Z}_m \setminus \{0\}$ exactly twice. For m odd, many procedures are available for constructing power-sequence terraces for $\mathbb{Z}_m$; each terrace of this sort may be partitioned into segments one of which contains merely the zero element of $\mathbb{Z}_m$ whereas each other segment is either (a) a sequence of successive powers of an element of $\mathbb{Z}_m$ or (b) such a sequence multiplied throughout by a constant. We now extend this idea by using power-sequences in $\mathbb{Z}_n$, where n is an odd prime, to obtain terraces for $\mathbb{Z}_m$ where $m = n + 2$. We provide $\mathbb{Z}_{n+2}$ terraces for all odd primes n satisfying $0 < n < 1000$ except for $n = 127$, 601, 683.

Partitioning a graph into two pieces, each isomorphic to the other or to its complement

M. Priesler (Moreno)

MSC2000: 05C60

A simple graph G has *the generalized-neighbour-closed-co-neighbour* property, or is *a gncc graph*, if for all vertices x of G, the subgraph, induced by the set of neighbours of x, is isomorphic to the subgraph, induced by the set of non-neighbours of x, or is isomorphic to its complement. If every vertex x satisfies the first condition (that is, the subgraphs, induced by its set of neighbours, and by its set of non-neighbours, are isomorphic), then the graph has *the neighbour-closed-co-neighbour* property, or is *an ncc graph.* The *ncc* graphs were characterized by A. Bonato and R. Nowakowski, and a polynomial time algorithm was given for their recognition. In this paper we show that all *gncc* graphs are also *ncc*, that is, we prove that the two families of graphs, defined above, are identical. Finally, we present some of the properties of an interesting family of graphs, that is derived from the proof of the claim above, and we give a polynomial time algorithm to recognize such graphs.

k-pseudosnakes in n-dimensional hypercubes

Erich Prisner

MSC2000: 05C69

A k-pseudosnake in a graph is an induced subgraph of maximum degree at most k. In this paper we show that k-pseudosnakes with more than 2^{n-1} vertices exist in the hypercubes Q_n, provided $n \leq 2k$. We also give upper bounds, and show that the generated k-pseudosnakes are maximum provided k is even and $n = 3k/2$. The results also yield better constructions of k-pseudosnakes in large n-dimensional grids in certain cases.

Local nature of Brooks' colouring

T.J. Rackham

MSC2000: 05C15

Brooks' theorem gives the existence of a $\Delta(G)$-colouring of a connected graph G when it is neither a complete graph nor an odd cycle. For such a Brooks' graph G with $\Delta(G) \geq 3$, we consider the problem of precolouring k vertices, where $k < \Delta(G)$, and ask whether this can be extended to a proper $\Delta(G)$-colouring of all of G. We have shown that this can always be done if the vertices being precoloured are mutually a distance at least 6 apart in G, and this bound is tight. This result improves a result of Sajith and Saxena, who showed that a sufficient distance exists in maximum degree 3 graphs; and will be seen to complement work of Axenovich, and of Albertson et al, who independently gave a sufficient distance of 8 for precolouring any size of an independent set of vertices.

We will outline the method of proof of this result, which differs significantly for graphs of maximum degree 3 to those of higher maximum degree. We also give the extremal counterexamples for distance 5.

On bicyclic reflexive graphs

Zoran Radosavljević

(joint work with Bojana Mihailović and Marija Rašajski)

MSC2000: 05C50

A simple graph is reflexive if the second largest eigenvalue of its $(0, 1)$-adjacency matrix does not exceed 2. By this paper we go on with the investigations initiated by the article "Which bicyclic graphs are reflexive?" (Z. Radosavljević, S. Simić, 1996), and continued in the meantime through considering some other classes of reflexive graphs. Former results mainly concern so-called treelike graphs or cactuses, i.e. graphs whose all cycles are mutually edge-disjoint. Provided that one cannot test whether a cactus is reflexive by removing a single cut-vertex, and that all its cycles do not have a common vertex, it turned out that such a graph has at most five cycles. Based on this fact and these two assumptions, it was possible to find all maximal reflexive cactuses with five and four cycles and to recognize some important facts concerning tricyclic reflexive cactuses, including the construction of some particular classes. These results also enabled perceiving some classes of bicyclic reflexive cactuses.

In this paper we present four new classes of maximal bicyclic reflexive graphs. One is constructed by substituting free cycles (those having only one vertex of degree $d > 2$) in tricyclic cactuses by Smith trees (trees whose index is $\lambda_1 = 2$). The other is also constructed starting from a characteristic class of tricyclic cactuses, but being generated by "pouring" of a triple of Smith trees between two characteristic vertices. The third class provides starting from a pair of free cycles with the common vertex of degree 5. Finally, one class is generated by θ-graphs (bicyclic graphs obtained by joining two vertices by three disjoint paths). At some stages the work has been supported by using the expert system GRAPH.

One-factorizations of the complete graph with a prescribed automorphism group

Gloria Rinaldi

MSC2000: 05C70, 05C75

The number of non-isomorphic one-factorizations of the complete graph K_{2n} explodes as n increases and a general classification is not possible. An attempt can be done if one imposes additional conditions on the automorphism group of the one-factorization. In this talk I focalize my attention on the following question:

For which groups G of even order $2n$ does a one-factorization of the complete graph K_{2n} exist with the property of admitting G as a sharply vertex transitive automorphism group?

When n is odd, G must be the semi-direct product of Z_2 with its normal complement and G always realizes a one-factorization of K_{2n} upon which it acts sharply transitively on vertices.

When n is even, the complete answer is still unknown. If G is a cyclic group the answer to the question is negative when n is a power of 2 greater than 4, while it is affirmative for all other values of n (Hartman and Rosa 1985). It is also affirmative if G is abelian and not cyclic (Buratti 2001), and if G is dihedral (Bonisoli and Labbate 2002).

I discuss other classes of groups.

Independent sets in extremal strongly regular graphs

P. Rowlinson

MSC2000: 05C50

Regular graphs with an eigenvalue μ of maximal multiplicity ($\mu \neq 0, -1$) are precisely the extremal strongly regular graphs. To within complements, only three such graphs are known. If G is such a graph then, replacing G with $\overline{G}$ if necessary, we may assume that $\mu > 0$. Then the independence number of G is at most $4\mu^2 + 4\mu - 2$, with equality if and only if G is one of the three known examples.

Orbits of graph automorphisms on proper vertex colourings

J.D. Rudd

(joint work with P.J. Cameron and B. Jackson)

MSC2000: 05C15, 05C25, 20B25

We use the orbital Tutte polynomial as defined by P.J. Cameron to count the number of orbits of the automorphism group of a connected graph Γ on proper vertex colourings of Γ from k colours. We then modify the orbital Tutte polynomial so that we can count orbits of the automorphism group on proper k-colourings for a disconnected graph.

Coprime polynomials over $GF(2)$

C.G. Rutherford

(joint work with R.W. Whitty)

MSC2000: 11C08, 11T06, 15A33

Corteel, Savage, Wilf and Zeilberger, (*JCT*, A, 82, 186-192, 1998) showed that exactly half of the ordered pairs of monic polynomials of degree n over $GF(2)$ are relatively prime pairs. They asked for a bijective proof of this fact. We build a table of resultant matrices and compare this to the addition table for $GF(2^n)$ (in which exactly half the entries are congruent to zero mod 2). This allows us to restate the problem in terms of pairs of subspaces of dimension 2 of $GF(2)^n$.

Deletion-similarity versus similarity of edges in graphs with few edge-orbits

G. Sabidussi

(joint work with L.D. Andersen and P.D. Vestergaard)

MSC2000: 05C60

Two edges e, e' of a graph G are *deletion-similar* if the edge-deleted subgraphs G_e and $G_{e'}$ of G are isomorphic (where $V(G_e) = V(G), E(G_e) = E(G) \setminus \{e\}$). Deletion similarity partitions $E(G)$ into equivalence classes called *deletion classes*. Trivially, if e and e' are in the same orbit with respect to Aut G then any automorphism of G mapping e to e' is an isomorphism of G_e onto $G_{e'}$. Hence deletion classes are unions of orbits. When do deletion classes and orbits coincide?

It has been shown that if $E(G)$ consists of one or two deletion classes, then deletion similarity implies similarity. On the other hand, for any $k \geq 5$ it is easy to construct graphs with exactly k deletion classes and more than k orbits. The present paper deals with the question of equality of deletion classes and orbits in graphs with exactly three deletion classes. We have not been able to give a complete answer, as the statement of the following theorem will make clear:

Theorem: *Let G be a graph with exactly three deletion classes of edges. Then these classes are orbits except possibly when G is obtained by deleting an edge from a Moore graph or a bipartite Moore graph (incidence graph of a projective plane).*

The case of graphs with exactly four deletion classes remains open.

Self-complementary two-graphs and almost self-complementary double covers over complete graphs

Mateja Šajna (joint work with Primož Potočnik)

MSC2000: 05C25, 05C65, 05C70

Let X be a graph of even order and $\mathcal{I}$ a 1-factor of the complement X^{C} of X. Then X is called *almost self-complementary (ASC)* with respect to the 1-factor $\mathcal{I}$ if it is isomorphic to its *almost complement* $X^{\mathrm{C}} - \mathcal{I}$. ASC graphs were introduced by Alspach as an analogue to self-complementary graphs for (regular) graphs of even order. ASC circulant graphs were first studied by Dobson and Šajna (2004), and general ASC graphs by Potočnik and Šajna (submitted). These papers revealed the complexity of the problem of ASC graphs: while every automorphism of a graph is also an automorphism of its complement, an automorphism of an ASC graph need not preserve the "missing" 1-factor. An automorphism of an ASC graph, as well as an isomorphism from an ASC graph to an almost complement, is called *fair* if it preserves the associated 1-factor. An ASC graph is called *homogeneously almost self-complementary (HASC)* if it admits a vertex-transitive group of fair automorphisms and a fair isomorphism into the almost complement that normalizes it. While general ASC graphs correspond to symmetric index-2 isomorphic factorizations of the graphs $K_{2n} - nK_2$, HASC graphs occur as factors of symmetric index-2 homogeneous factorizations of these graphs. (Homogeneous factorizations were introduced by Li and Praeger, and HASC graphs were recently studied by Potočnik and Šajna.) An HASC graph is called *2-transitively almost self-complementary* if its group of fair automorphisms acts 2-transitively on the edge set of the associated 1-factor. An ASC graph that is a double cover over a complete graph is called an *ASC double cover* if it is ASC with respect to a set of fibres. Similarly we define *HASC double covers*. A *two-graph* on a set Ω is a set $\mathcal{T}$ of unordered triples of points of Ω with the property that any unordered quadruple of points contains an even number of triples in $\mathcal{T}$. Two-graphs were introduced by Higman in the 1970s, and later studied by Taylor.

In the main result of this talk we shall describe a one-to-one correspondence between the isomorphism classes of self-complementary two-graphs and ASC double covers, vertex-transitive self-complementary two-graphs and HASC double covers, and 2-transitive self-complementary two-graphs and 2-transitively ASC graphs. From this correspondence and Tayor's classification of 2-transitive two-graphs it follows that there exists (up to isomorphism) a unique 2-transitively ASC graph of every admissible order.

On the number of independent sets in graphs

Alexander Sapozhenko

MSC2000: 05C69

We improve our previous upper bounds [4] for the number of independent sets in graphs. Similar bounds turn out to be useful in solving some combinatorial problems of the group theory and the number theory (see for example, [1], [2], [3]). The new bounds have the form $I(G) \leq 2^{(p/2)(1-\varepsilon)}$, where $I(G)$ is the number of independent sets of graph G, p is the number of its vertices, and ε is a positive constant depending on G.

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An additive structure of BIB designs

Masanori Sawa

(joint work with H. Kiyama, D. Matsumoto, K. Matsubara, S. Kageyama)

MSC2000: 51B05, 62K10

Does there exist a set of s BIBD($v = sk$, $b = sr$, r, k, λ) with s incidence matrices $\mathbf{N}_i$, $i = 1, \ldots, s$, which satisfies the following two conditions

(1) $\sum_{i=1}^{s} \mathbf{N}_i = J$, where J is a matrix of size $v \times b$, all whose elements are zero,

(2) $\mathbf{N}_{i_1} + \mathbf{N}_{i_2}$ is the incidence matrix of a BIBD($v^* = sk$, $b^* = sr$, $r^* = 2r$, $k^* = 2k$, λ^*) for any distinct $i_1, i_2 \in \{1, \ldots, s\}$?

We say such BIB designs have an *additive structure*. In this talk, direct and recursive constructions of BIB designs having an additive structure are discussed. Characterizations of parameters of such structures are also given.

On monophonic sets in graphs

Carlos Seara

(joint work with Carmen Hernando, Mercè Mora and Ignacio M. Pelayo)

MSC2000: 05C12, 05C05

We deal with two types of graph convexities, which are defined by a system $\mathcal{P}$ of paths in a connected graph $G = (V, E)$: the geodetic convexity (also called the metric convexity)[3, 4] which arises when we consider shortest paths, and the monophonic convexity (also called the minimal path convexity)[2, 3] when we consider chordless paths. Given G and two vertices u, v in V, a chordless $u - v$ path in G is called a $u - v$ monophonic path. Let $J[u, v]$ denote the set of all vertices in G lying on some $u - v$ monophonic path. Given a set $S \subseteq V$, let $J[S] = \bigcup_{u,v \in S} J[u, v]$. If $J[S] = V$, then S is called a monophonic set of G. If $J[S] = S$, then S is called a m-convex set of G. The monophonic convex hull $[S]_m$ of S is the smallest m-convex set containing S. If $[S]_m = V$, then S is called a m-hull set of G. If we restrict ourselves to shortest paths, we obtain the geodetic and g-hull sets, which have been widely studied in the recent years.

We study monophonic sets in a connected graph G. Firstly, we present a realization theorem proving that there is no general relationship between monophonic and geodetic hull sets. Second, we study the contour of a graph [1] (a generalization of the set of extreme vertices) showing that the contour of G is a monophonic set. Finally, we focus our attention on the edge Steiner sets. We prove that every edge Steiner set S in G is edge monophonic, i.e., every edge of G lies on some monophonic path joining two vertices of S.

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Permutations and Quantum Entanglement

S. Severini

(joint work with L. Clarisse, S. Ghosh, A. Sudbery)

MSC2000: 81P68, 11G20

Entanglement is a fundamental notion in quantum mechanics. Recently, the advent of quantum information theory and quantum computation has highlighted the role of entanglement as a resource in many applications including fast algorithms and classically secure cryptographic protocols. The notion of *entangling power* of unitary matrices was introduced by Zanardi, Zalka and Faoro [*Physical Review A*, **62**, 030301]. We study the entangling power of permutations (that is, of permutatiom matrices), given in terms of a combinatorial formula. We characterize the permutation with zero entangling power. We construct the permutations with the minimum nonzero entangling power for every dimension. With the use of orthogonal latin squares, we construct the permutations with the maximum entangling power for every dimension. Moreover, we show that the value obtained is maximum over all unitary matrices of the same dimension, with possible exception for 36. We numerically classify, according to their entangling power, the permutations of length 4 and 9, and we give some estimates for longer lengths. This work suggests a number of open problems of combinatorial nature concerning random matrix theory, error-correcting codes, expander graphs, *etc.* The talk is mainly based on xxx.soton.ac.uk/abs/quant-ph/0502040

Grassmann and Segre varieties over GF(2): some graph theory links

R. Shaw

MSC2000: 51E20, 05C30, 05C90, 14G25

Consider:

(i) the Grassmann variety $\mathcal{G}_{1,n,2}$ of the lines of PG(n,2), a subset of the finite projective space $\mathrm{PG}(\binom{n+1}{2}-1,2)=\mathbb{P}(\wedge^2 V_{n+1,2})$;

(ii) the Segre variety $\mathcal{S}_{m,n,2}$, a subset of the finite projective space $\mathrm{PG}(mn+m+n,2)=\mathbb{P}(V_{m+1,2}\otimes V_{n+1,2})$.

In the case of (i) results (and a conjecture) concerning the polynomial degree of $\mathcal{G}_{1,n,2}$ have recently been obtained (see R. Shaw and N.A. Gordon, (2005), The polynomial degree of the Grassmannian $\mathcal{G}(1,n,2)$, accessible from: http://www.hull.ac.uk/maths/people/rs/staffdetails.html).
These are shown to be equivalent to results (and a conjecture) concerning certain kinds of subgraphs of those (simple) graphs $\Gamma=(\mathcal{V},\mathcal{E})$ which are of order $|\mathcal{V}|=n+1$. It turns out that those graphs Γ of size $|\mathcal{E}|=n=|\mathcal{V}|-1$ are of particular significance.

In the case of (ii) it is shown that results concerning the polynomial degree of $\mathcal{S}_{m,n,2}$ are equivalent to the following assertions concerning certain subgraphs of any bipartite graph $\Gamma=(\mathcal{V},\mathcal{E})$ whose parts have sizes $m+1$ and $n+1$.

> Let $N(\Gamma)$ denote the total number of subgraphs of Γ which are isomorphic to the complete bipartite graph $\Gamma_{m'+1,n'+1}$ for some m' and n' satisfying $0<m'\leq m$ and $0<n'\leq n$. In the cases $m\leq n$ the following hold:
>
> (a) if $|\mathcal{E}|>mn+m$ then $N(\Gamma)$ is odd for all such bipartite graphs Γ;
>
> (b) if $|\mathcal{E}|\leq mn+m$ then $N(\Gamma)$ is even for some such bipartite graph Γ.

Mendelsohn 3-frames and embeddings of resolvable Mendelsohn triple systems

Hao Shen

MSC2000: 05B07

In this talk we will determine necessary and sufficient conditions for the existence of Mendelsohn 3-frames. We will also determine necessary and sufficient conditions for the embeddings of resolvable Mendelsohn triples and embeddings of almost resolvable Mendelsohn triple systems.

Constructing linear codes from some orbits of projectivities

M. Shinohara

(joint work with T. Maruta and M. Takenaka)

MSC2000: 94B05, 94B15, 51E20

We denote by F_q the field of q elements. Let $g(x)$ be a monic polynomial of degree k in $F_q[x]$ and let T be the companion matrix of $g(x)$. Let τ be the projectivity of $\mathrm{PG}(k-1, q)$ defined by T with order N. We define an $[mN, k]_q$ code $\mathcal{C}$ from m orbits of τ and we show that $\mathcal{C}$ is a degenerate quasi-twisted code. A lot of new linear codes over the field of q elements ($q \leq 9$) are found from such codes by some combinations of puncturing or extending.

Some new results on the index of trees

S.K. Simić

(joint research with: F. Belardo, E.M. Li Marzi, D.V. Tošić and B. Zhou)

MSC2000: 05C50

We identify those trees whose index (the largest eigenvalue of the adjacency matrix) is maximal in the case that:

(1) the largest (vertex) degree is prescribed;

(2) the diameter is prescribed along with some other structural details.

We also identify in the set of trees having diameter d the tree with the k-th largest index, where $k = 1, \dots, \lfloor \frac{d}{2} \rfloor$.

Maximal nontraceable graphs of small size

J.E. Singleton

(joint work with M. Frick)

MSC2000: 05C38

A graph G is *maximal nontraceable* (MNT) if G is not traceable, i.e. if G does not contain a Hamiltonian path, but $G + e$ does contain a Hamiltonian path for all $e \in E(\overline{G})$.

Most constructions for MNT graphs in the literature (see [1], for example) depend on large cliques, thus yielding fairly dense graphs. To date, no cubic MNT graphs have appeared in the literature.

We construct an infinite family of 2-connected cubic MNT graphs and show that, for all even $n \geq 50$ the lower bound for the size of a 2-connected graph of order n equals $\frac{3n}{2}$.

Recently, Dudek, Katona and Wojda showed that for $n \geq 20$ every MNT graph of order n has size at least $\lceil \frac{3n-2}{2} \rceil - 2$ and for each $n \geq 54$ as well as for $n \in I = \{22, 23, 30, 31, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51\}$ they constructed a MNT graph of order n and size $\lceil \frac{3n-2}{2} \rceil$.

We establish the exact lower bound for the size of a MNT graph of order n, for $n \geq 54$ and $n \in I$, as well as for $n \leq 10$ and $n = 12, 13$.

Reference.

[1] B. Zelinka, *Graphs maximal with respect to absence of hamiltonian paths*, Discuss. Math. Graph Theory **18** (1998), 205-208.

Factorisation of snarks

Martin Škoviera

(joint work with Miroslav Chladný)

MSC2000: 05C15, 05C75

We develop a theory of factorisation of snarks — cubic graphs with edge-chromatic number 4 — based on the classical concept of the dot-product. Our main concern are *irreducible* snarks, those where the removal of every non-trivial edge-cut yields a 3-edge-colourable graph. We show that if an irreducible snark can be expressed as a dot-product of two smaller snarks, then both of them are irreducible. This result constitutes the first step towards the proof of the following "unique-factorisation" theorem:

Every irreducible snark G *can be factorised into a collection* $\{H_1, \ldots, H_n\}$ *of cyclically* 5*-connected irreducible snarks such that* G *can be reconstructed from them by iterated dot-product. Moreover, such a collection is unique up to isomorphism and ordering of the factors regardless of the way in which the decomposition was performed.*

The result is best possible in the sense that it fails for snarks that are close to being irreducible but themselves are not irreducible.

The unique-factorisation theorem can be extended to the case of factorisation with respect to a preassigned subgraph K which is required to stay intact during the whole factorisation process. We show that if K has order at least 3, then the theorem holds, but is false when K has order 2.

Cyclically permutable codes and simplex codes

Derek H. Smith

(joint work with Stephanie Perkins)

MSC2000: 94B05, 94B15

A *cyclically permutable code* is a binary block code of length n such that each codeword has n distinct cyclic shifts and such that no codeword can be obtained by one or more cyclic shifts of another codeword.

The usual constructions of cyclically permutable codes start from a cyclic code and select one codeword from each cyclic equivalence class of full order. In this talk code equivalence is used to construct cyclically permutable simplex codes when they exist. The construction extends to show that certain cyclic codes are equivalent to cyclically permutable codes. In this way larger codes are obtained. An application to code-division multiple-access is given, and methods of increasing the cyclic minimum distance are presented.

Vertex-distinguishing proper edge colouring of some regular graphs

Roman Soták

(joint work with Janka Rudašová)

MSC2000: 05C15

A proper edge colouring of a simple graph G is called *vertex-distinguishing* if no two distinct vertices have the same set of colours of their incident edges. The minimum number of colours in such colouring (if it exists at all) is denoted by $\chi'_s(G)$. Burris and Schelp made the following conjecture: Let G be a graph with no isolated edges and with at most one isolated vertex. Let k be the minimum integer such that $\binom{k}{d} \geq |\{v : deg_G(v) = d\}|$ for all d with $\delta(G) \leq d \leq \Delta(G)$. Then $\chi'_s(G) \in \{k, k+1\}$.

In this talk this conjecture is proved for some r-regular graphs with only small components. Moreover it is proved that any graph G can be given a vertex-distinguishing equitable proper edge colouring by k colours for any $k \geq \chi'_s(G)$. Here equitable means that cardinalities of any two distinct colour classes differ by at most 1.

Random preorders

Dudley Stark

(joint work with Peter Cameron)

MSC2000: 05A16, 05C83

A random preorder on n elements consists of linearly ordered equivalence classes called *blocks*. We investigate the block structure of a preorder chosen uniformly at random from all preorders on n elements as $n \to \infty$. Time permitting, related work on random 0-1 matrices with Peter Cameron and Thomas Prellberg may be discussed.

Defining sets of full designs and other simple designs

Anne Penfold Street

(joint work with Ken Gray, Colin Ramsay and Emine Şule Yazıcı)

MSC2000: 05B05, 05B07, 05B99

A set of blocks which is a subset of a unique t-(v, k, λ_t) balanced incomplete block design (*BIBD*) is a *defining set* of the design. A *full* design is a simple *BIBD* comprising all k-tuples on a given set of v elements. We present results on their defining sets which are often useful, despite their relatively large λ values, since we show that a defining set of any simple *BIBD* can often be derived from a defining set of the corresponding full design.

Minimal claw-free graphs

Henda C. Swart

(joint work with P.A. Dankelmann, W.D. Goddard, M.D. Plummer and P. van den Berg)

MSC2000: 05C75

A graph G is a minimal claw-free graph (MCFG) if it contains no $K(1,3)$ (claw) as induced subgraph and if, for each edge e of G, $G-e$ contains an induced claw. We investigate properties of MCFGs, establish sharp bounds on their orders and the degree of their vertices, characterize graphs which have minimally claw-free line graphs and find bounds on the order, vertex degrees and connectivity of MCFGs which have independence number equal to 2.

Contractible digraphs, fixed cliques, and the Cop-robber games

Rueiher Tsaur

MSC2000: 05C20, 05C75

A most interesting recent development in the study of dismantlable (undirected) graphs is that dismantlable graphs have turned out to be significant for discrete physical modelling (G.R. Brightwell and P. Winkler, Gibbs measures and dismantlable graphs, J. Combin. Theory Ser. B, 78:141–166, 2000). It is noteworthy that non-reflexive graphs are needed in this work, whereas all previous studies of dismantlability have assumed reflexivity. In this presentation, a non-recursive definition of "dismantlability" for (reflexive or not) digraphs is introduced, thus providing a firm foundation for such work. We show that this definition extends and unifies various definitions of dismantlable structures.

In the remainder of the presentation, we aim to extend the notions of fixed clique and point properties and cop-robber games, from undirected graphs to digraphs, with special emphasis on dismantlable digraphs.

Dominating sequences and traversals of ordered trees

P.-G. Tsikouras

(joint work with I. Tasoulas)

MSC2000: 05C05, 05C07

An ordered tree with root r is a triplet $T = (V, \Gamma, l)$ where $r \in V$, $\Gamma : V \setminus \{r\} \to V$ and $l : V \to \mathbb{N}^*$ such that $l(r) = 1$, $l(x) = l(\Gamma(x)) + 1$ for every $x \in V \setminus \{r\}$ and the sets $\Gamma^{-1}(\{x\})$ are totally ordered.

Four basic ways of traversing ordered trees (level order, preorder, postorder, inorder) are studied in this context.

While in each of the first three cases the respective degree sequence determines uniquely the ordered tree, we realize that this is not true in the inorder case.

The degree sequences of the ordered trees according to each traversal are related to dominating sequences; in particular the inorder degree sequence is dominating.

So, for every dominating sequence, in the three first cases we present constructions of the corresponding unique ordered tree, whereas in the inorder case we construct recursively all the corresponding ordered trees.

Chordal double bound graphs and posets

Morimasa Tsuchiya

(joint work with H. Era, S.-I. Iwai and K. Ogawa)

MSC2000: 05C62

We consider properties of double bound graphs with respect to subposets. The *double bound graph* (*DB-graph*) of $P = (X, \leq_P)$ is the graph $DB(P) = (X, E_{DB(P)})$, where $xy \in E_{DB(P)}$ if and only if $x \neq y$ and there exist $m, n \in X$ such that $n \leq_P u, v \leq_P m$. We already know that for a graph G, there exists a double bound graph which contains G as an induced subgraph. We introduce a concept of (n, m)-subposets and obtain the next result.

Proposition 1 *For a poset P and a subposet Q of P, Q is an (n, m)-subposet of P if and only if $DB(Q)$ is an induced subgraph of $DB(P)$.*

Based on this result, we deal with poset theoretical properties of cycle graphs C_n and path graphs P_n and obtain the following result.

Theorem 2 *For a poset P, $DB(P)$ is a chordal graph if and only if (1) the induced subposet $\langle Max(P) \cup Min(P) \rangle_P$ does not contain Q_n ($n \geq 2$) as an induced subposet, and (2) d_can(P) does not contain $\{\delta\} \oplus Q_n$ and $Q_n \oplus \{\delta\}$ ($n \geq 4$) as an (n, m)-subposet.*

Furthermore we deal with properties of posets whose double bound graph is isomorphic to its upper bound graph, or its comparability graph, etc.

Balanced C_4-quatrefoil designs

K. Ushio

MSC2000: 05B30, 05C70

In graph theory, the decomposition problem of graphs is a very important topic. Various type of decompositions of many graphs can be seen in the literature of graph theory.

Let K_n denote *the complete graph* of n vertices. *The complete multi-graph* λK_n is the complete graph K_n in which every edge is taken λ times. Let C_4 be *the* 4*-cycle* (or *the cycle* on 4 vertices). *The C_4-quatrefoil* is a graph of 4 edge-disjoint C_4's with a common vertex and the common vertex is called *the center of the C_4-quatrefoil*.

When λK_n is decomposed into edge-disjoint sum of C_4-quatrefoils, we say that λK_n *has a C_4-quatrefoil decomposition.* Moreover, when every vertex of λK_n appears in the same number of C_4-quatrefoils, we say that λK_n *has a balanced C_4-quatrefoil decomposition* and this number is called *the replication number.* This balanced C_4-quatrefoil decomposition of λK_n is to be known as *a balanced C_4-quatrefoil design.*

We show that the necessary and sufficient condition for the existence of such a balanced C_4-quatrefoil design is $\lambda(n-1) \equiv 0 \pmod{32}$ and $n \geq 13$.

Claw-free graphs with non-clique μ-subgraphs and related geometries

I.A. Vakula

(joint work with V.V. Kabanov)

MSC2000: 05C75, 51E14

We describe finite ordinary connected claw-free graphs that contain a 3-coclique, in which every pair of vertices at distance two lies in induced 4-cycle. We also define a class of partial geometries with lines of cardinality two and three such that complements of their collinearity graphs satisfy conditions above.

New results on the Zarankiewicz Problem

J.C. Valenzuela

(joint work with C. Balbuena, P. García-Vázquez and X. Marcote)

MSC2000: 05C35, 05C40.

Let (X, Y) denote a bipartite graph with classes X and Y such that $|X| = m$ and $|Y| = n$. A complete bipartite subgraph with s vertices in X and t vertices in Y is denoted by $K_{(s,t)}$.

The Zarankiewicz problem consists in finding the maximum number of edges, denoted by $z(m, n; s, t)$, of a bipartite graph (X, Y) with $|X| = m$, $|Y| = n$, and without a complete bipartite $K_{(s,t)}$ as a subgraph. This problem is related with a Turán problem for bipartite graphs. Let us denote by $ex(m, n; K_{s,t})$ the maximum number of edges in a bipartite graph (X, Y) with $|X| = m$, $|Y| = n$, and free of $K_{s,t}$, that is to say, without both $K_{(s,t)}$ and $K_{(t,s)}$ as subgraphs. First we present a new upper bound for both extremal functions $z(m, n; s, t)$ and $ex(m, n; K_{s,t})$, which is attained if $\max\{m, n\} \leq s + t - 1$. Then we characterize the family $Z(m, n; s, t)$ of extremal graphs with size $z(m, n; s, t)$ for the values of the parameters described above.

Besides, new lower bounds for the Zarankiewicz function $z(m, n; s, t)$ are given for several cases. Additionally, this lower bound is proved optimum if $t \leq m \leq n = 2t$.

Maximal non-traceable oriented graphs

S.A. van Aardt

(joint work with M. Frick, J. Dunbar and O. Oellermann)

MSC2000: 05C20, 05C38

An oriented graph D is called *traceable* if there is a directed path in D that visits every vertex of D. A nontraceable oriented graph D is called *maximal non-traceable* (MNT) if $D + uv$ is traceable for every pair u, v of nonadjacent vertices in D.

We characterize the acyclic and the unicyclic MNT oriented graphs as well as the strong component digraphs of MNT oriented graphs. This enables us to characterize MNT oriented graphs of order n that have size $\binom{n}{2} - 1$ and we show that no MNT oriented graph of order n has size $\binom{n}{2} - 2$. We also show that the maximum size of a *strong* MNT oriented graph of order n is $\binom{n}{2} - 3$.

The number of edges in a bipartite graph of given order and radius

P. van den Berg

(joint work with P.A. Dankelmann and Henda C. Swart)

MSC2000: 05C12

Vizing established an upper bound on the size of a graph of given order and radius. We find sharp bounds on the size of a bipartite graph of given order and radius.

Difference families arising from infinite translation designs

A. Vietri

MSC2000: 05B10, 05B30

If we consider the difference set $\{0,1,3\}_{(mod\ 7)}$ and the difference family $\{\,\{0,1,4\},\{0,2,7\}\,\}_{(mod\ 13)}$, then a subtle *difference* between them may be observed. Namely, the former owes its algebraic success merely to $\mathbf{Z}$, and not to $\mathbf{Z}_7$ (indeed, $\Delta\{0,1,3\} = \{\pm1,\pm2,\pm3\}$) whereas the latter is indebted to $\mathbf{Z}_{13}$ for magically transforming ±7 into ∓6, thus filling the gap which did not look so nice in $\mathbf{Z}$ (indeed, $\Delta\{\,\{0,1,4\},\{0,2,7\}\,\} = \{\pm1,\pm2,\pm3,\pm4,\pm5,\pm7\}$). The elegant behaviour of difference sets (or families) like $\{0,1,3\}$ can be easily rephrased in the 2-dimensional context, that is, in $\mathbf{Z}\times\mathbf{Z}$.

Definition. A set A made up of 3-subsets of $\mathbf{Z}\times\mathbf{Z}$ is a *perfect d-family* if $\Delta A = [-d,d]\times[-d,d]\setminus\{(0,0)\}$.

Perfect d-families are – as it may be expected – quite generous creatures, as they unselfishly provide us with actual difference families, once projected upon $\mathbf{Z}_{2d+1}\times\mathbf{Z}_{2d+1}$. Recalling that a major concern for a number of mathematicians is to give birth to infinitely many somethings in a single hit, the following result can now be administered.

Theorem. For every $n \geq 0$ there exists a perfect, not pure, $12\cdot2^n$-family. Furthermore, for every $n \geq 0$ there exists a perfect, not pure, $20\cdot4^n$-family. Finally, there exists a perfect, not pure, 8-family.

What is a *pure* family? Why demanding even more from an already perfect being? Because of Measure Theory. Alas, in the 1-dimensional environment we were secluded in a scanty line, and could behold nothing but thready, degenerate triangles! In the 2-dimensional context things change, for we can now distinguish a degenerate triangle from a nondegenerate one. Consequently, we might not be contented with a family of blocks of size 3 some of which are degenerate. Perhaps we would welcome a family entirely consisting of nondegenerate triangles, that is, what we call a pure family.

Is pureness an utopia? Certainly not, because two pure d-families can be exhibited when $d = 3$ and $d = 8$.

Combinatorial algorithm for finding a clique of maximum weight in a C_4-free Berge graph

Kristina Vušković

(joint work with Gérard Cornuéjols)

MSC2000: 05C85, 05C17, 90C27, 68R10, 68Q25

A *hole* is a chordless cycle of length at least four. A graph is *Berge* if it does not contain (as an induced subgraph) an odd hole nor a complement of an odd hole. A graph G is *perfect* if every induced subgraph H of G satisfies $\chi(H) = \omega(H)$, where $\chi(H)$ denotes the chromatic number of H and $\omega(H)$ denotes the size of a largest clique in H. In 2002 Chudnovsky, Robertson, Seymour and Thomas proved the famous Strong Perfect Graph Conjecture (SPGC), posed by Berge in 1961, that states that a graph is perfect if and only if it is Berge. Moreover, in 2003, Chudnovsky, Cornuéjols, Liu, Seymour and Vušković gave a polynomial time recognition algorithm for Berge graphs.

One important aspect of perfect graphs is that the following optimization problems: maximum weighted clique, maximum weighted stable set, minimum weighted covering of vertices by cliques and minimum weighted covering of vertices by stable sets, that are NP-complete in general, can be solved in polynomial time for perfect graphs. This was shown by Grötschel, Lovász and Schrijver in the 80's. Their algorithm uses the ellipsoid method and a polynomial time separation algorithm for a certain class of positive semidefinite matrices related to Lovász' upper bound on the Shannon capacity of a graph. The question remains whether these four optimization problems can be solved for perfect graphs by polynomial time purely combinatorial algorithms, avoiding the ellipsoid method.

A C_4-free Berge graph is a Berge graph that does not contain, as an induced subgraph, a hole of length 4. The aim of this paper is to provide a combinatorial $O(n^9)$-time algorithm that computes a maximum weighted clique for every C_4-free Berge graph.

This algorithm is decomposition based, but interestingly it does not use any of the previously known decomposition theorems for C_4-free Berge graphs. (C_4-free Berge graphs were first studied by Conforti, Cornuéjols and Vušković, in 2000, when they obtained a decomposition theorem for this class that they used to prove the SPGC for C_4-free graphs. Later Chudnovsky, Robertson, Seymour and Thomas obtained a decomposition theorem, of similar flavor, for Berge graphs that they used to prove the SPGC in general.) Maximum weighted clique is computed by decomposing a C_4-free Berge graph using "full star decompositions" into triangulated graphs.

Tabu search for Covering Arrays using permutation vectors

Robert A. Walker II

(joint work with Charles J. Colbourn)

MSC2000: 05B15, 05B40, 68T20

A *covering array* $\mathrm{CA}(N;t,k,v)$ is an $N \times k$ array. In any $N \times t$ subarray, each possible t-tuple over v symbols (there are v^t of these) occurs at least one time. The parameter t is referred to as the *strength* of the array. Covering arrays have a wide range of applications including software interaction testing. A compact representation of certain covering arrays employs "permutation vectors" to encode $v^t \times 1$ subarrays of the covering array. Sherwood et al. (2005) have shown that a covering perfect hash family whose entries correspond to permutation vectors yields a covering array. We introduce a method for efficient search for covering arrays of this type using Tabu search. Using this technique, improved covering arrays of strength 3 and 4 have been found, as well as the first arrays of strength 5, 6, and 7 found by computational search.

The equitable colouring of planar graphs with large girth

Ping Wang

(joint work with J.L. Wu and Y.Z. Ni)

MSC2000: 05C15

A proper vertex-coloring of a graph G is equitable if the size of color classes differ by at most one. The equitable chromatic threshold of G, denoted by $\chi^*_{Eq}(G)$, is the smallest integer n such that G is equitably k-colorable for all $k \geq n$. We prove that $\chi^*_{Eq}(G) = \chi(G)$ if G is a planar with girth ≥ 16 and $\delta(G) \geq 2$ or G is a 2-connected non-bipartite outplanar with girth ≥ 4.

Perfect 1-factorisations and atomic Latin squares

I.M. Wanless

(joint work with Darryn Bryant and Barbara Maenhaut)

MSC2000: 05C70, 05B15

A *perfect* 1*-factorisation* (P1F) of a graph is a partition of the edge-set of that graph into 1-factors (perfect matchings) with the property that the union of any two of the 1-factors is a Hamiltonian cycle. Since the dawn of time two infinite families of P1Fs of complete graphs have been known. Recently we discovered a third family while in pursuit of another type of elusive beast known as an *atomic latin square*. These are latin squares with an indivisible structure akin to that of the cyclic groups of prime order. In this talk I will discuss the P1Fs and latin squares that we found.

Some list colouring problems in the reals

R.J. Waters

MSC2000: 05C15

List colouring is a generalisation of ordinary graph colouring, in which the colour of each vertex must be chosen from a list of colours assigned to that vertex. We consider two variations of the list colouring problem where the 'lists' are subsets of the real line, and the colours assigned to adjacent vertices must differ by at least 1.

In the first of these two problems, each vertex of a graph G is assigned an interval of length k as its list. We introduce a new graph invariant $\tau(G)$, called the *consecutive choosability ratio* and defined to be the smallest k such that a colouring as described above can always be found. In the second problem, the lists are arbitrary closed sets of the real line of measure k, and the corresponding invariant $\sigma(G)$ is called the *choosability ratio*.

We present a selection of the results obtained to date regarding these parameters, including general bounds on $\tau(G)$ and $\sigma(G)$, values for specific classes of graphs, and relationships with other graph invariants such as the chromatic and list-chromatic numbers.

Representing $(d, 3)$-tessellations as quotients of Cayley maps

B.S. Webb

(joint work with J. Šiagiová)

MSC2000: 05C10

A $(d, 3)$-tessellation is a planar map all of whose vertices have valence d, and all of whose faces are triangles. Interest in $(d, 3)$-tessellations comes from triangulations—embeddings of simple graphs onto surfaces such that all the faces are triangles—since any triangular embedding is a quotient of a triangular tessellation.

To begin to answer the question

> In how many ways can a $(d, 3)$-tessellation be represented as a Cayley map?

we look at one-vertex quotient maps of degree d, all of whose faces are triangles.

On some stability theorems in finite geometry

Zsuzsa Weiner

(joint work with Tamás Szőnyi)

MSC2000: 51E21

By the stability of a point set H, we mean that every point set that is 'near' to H can be obtained from H by adding and deleting a 'few' points. The stabilty questions define the words 'near' and 'few' precisely for a given point set.

In Galois geometries combinatorially defined point sets with maximum/minimum cardinality are often nice in the sense that their intersection number with lines can only take up a few values. Usually easy combinatorial counting shows that a point set of size near to the extremal point set(s) can only have a very small number of lines with non-typical intersection number.

The algebraic method first used in [1], later improved in [2], can be used to show that the above non-typical lines should pass through a few number of points. Hence point sets with sizes close to the extremal ones can be obtained by adding and deleting a few points from the extremal point sets. Note that the first such theorem is *Segre's* embeddability theorem on arcs/hyperovals. Our method yields stability theorems on arcs, (k, n)-arcs, blocking sets and sets without tangents.

References.

[1] T. Szőnyi, On the embeddability of (k, p)-arcs, *Designs, Codes and Cryptography* **18** (1999), 235–246.

[2] Zs. Weiner, On (k, p^e)-arcs in Galois planes of order p^h, *Finite Fields and Appl.*, **10** (2004), 390–404.

Minimum dominating walks on graphs with large circumference

C.A. Whitehead

(joint work with B. L. Hartnell)

MSC2000: 05C69, 05C90

A *dominating walk* W in G is a walk such that for each $v \in V(G)$, either $v \in V(W)$ or v is adjacent to a vertex of W. The concept was introduced to model the situation of a security guard, or team of guards, monitoring a site in which each point of interest has to be visited or seen from a neighbouring point on a regular basis. In the case where there is just one guard, it is of interest to determine a closed dominating walk of minimum length in the graph representing the site to be monitored. Finding the length of such a walk in a general graph is known to be computationally difficult and exact values are known in the case of only a few special families.

We show how a closed minimum dominating walk may be obtained in two infinite families of graphs G containing a longest cycle C such that every vertex of $V(G) \setminus V(C)$ is of degree 2.

Rook polynomials on 2-dimensional surfaces

R. W Whitty

MSC2000: 05A05, 05A15, 05C78, 37F20

By a simple trick we may generalise the rook polynomial for an $n \times n$ chessboard to various 2-dimensional surfaces, the conventional chessboard corresponding to the torus. In the case of the Möbius band and the Klein bottle there is a close connection to graceful labellings of graphs. This connection can be exploited in calculating the rook polynomials.

Edge-bandwidth of grids and tori

Jerzy Wojciechowski

(joint work with Oleg Pikhurko)

MSC2000: 05C35, 05C78

The *edge-bandwidth* of a graph G is the bandwidth of the line graph of G, that is, it is the smallest number B' for which there is an injective labeling of $E(G)$ with integers such that the difference between the labels at any adjacent edges is at most B'.

We compute the edge-bandwidth for rectangular grids:

$$B'(P_m \oplus P_n) = 2\min(m,n) - 1, \quad \text{if } \max(m,n) \geq 3,$$

where $\oplus$ is the Cartesian product and P_n denotes the path on n vertices. This settles a conjecture of Calamoneri, Massini and Vrťo [*Theoret. Computer Science*, **307** (2003) 503–513].

We also compute the exact value of the edge-bandwidth of a product of two graphs $F \oplus P_n$ where F is a connected graph with $|E(F)| \leq n-1$, and of any torus (a product of two cycles) within an additive error of 5.

Recent results on total choosability and edge colourings

D.R. Woodall

MSC2000: 05C15

This talk is based on four papers: two submitted (one jointly with Tim Hetherington) and two in preparation.

The *total choosability* $\mathrm{ch}''(G)$ of a graph G is the smallest number k such that if every element (vertex or edge) of G is assigned a list of k colours, then every element can be coloured with a colour from its own list in such a way that every two adjacent or incident elements are coloured differently. The (ordinary) edge and total chromatic numbers of G are denoted by $\chi'(G)$ and $\chi''(G)$ respectively, and $\Delta(G)$ is the maximum degree of G.

It is proved that $\mathrm{ch}''(G) = \chi''(G) = \Delta(G) + 1$ if G is a series-parallel (K_4-minor-free) graph with $\Delta(G) \geq 3$; the hardest case (by far) is when $\Delta(G) = 3$. The same holds if G is $K_{2,3}$-minor-free, unless $\Delta(G) = 3$ and G has a K_4 component.

It is proved that $\mathrm{ch}''(G) = \chi''(G) = 4$ if $\Delta(G) = 3$ and every subgraph of G has average degree at most $2\frac{1}{2}$; this fills in a missing case in a result of Borodin, Kostochka and Woodall (1997).

A graph is *edge-k-critical* if $\Delta(G) = k$, $\chi'(G) > k$, and $\chi'(G - e) = k$ for each $e \in E(G)$. A new lower bound is obtained on the average degree of an edge-Δ-critical graph, which improves on the best bound previously known for almost all $\Delta \geq 4$.

On related combinatory problems in information cartography

Bilal Yalaoui

(joint work with Madjid Dahmane and Hacene Ait Haddadene)

MSC2000: 05C85, 05C90, 68R05, 68T30, 68U15

The concept of Information Cartography has evolved, elaborated and matured over time. It was not originally envisioned as a context-independent tool for visualizing and analyzing data/information/knowledge from practically any source.

In this context, graph's structures are the natural modelling support to do it. Here we consider the information cartography as textual corpus mining analyzing tool. Staring from one textual corpus divided into selected finite set of textual units, the terms extraction techniques help as to dress a list of used terms and several statistical data. The existing works used the associated graph of terms to analyse and build the text mining cartography based on selected frequent terms and co-occurrence.

In this contribution we will show first that the usually used model may be modified to be an oriented net for more semantic preservation of text content. In the second step we will show how graph clustering techniques can be used in information cartography building. Thus, variants of vertices density and edges force clustering graphs methods are given. And finally, we wil propose a new clustering technique based on graph triangularization and the graph clique partition.

Minimal homogeneous Steiner triple trades

E.Ş. Yazici

(joint work with N. Cavenagh and D. Donovan)

MSC2000: 05B07, 05B15

A Steiner triple trade (STT) is a subset of a Steiner triple system (STS) which may be replaced by a disjoint set of triples to create a new STS. An STT is called d-homogeneous if each point occurs in either 0 or d blocks of the trade. In this talk we give an existence proof of d-homogeneous Steiner triple trades for all $d \geq 3$.

Total domination in graphs

A. Yeo

(partially joint work with S. Thomasse)

MSC2000: 05C69, 05C65

A total dominating set S in a graph $G = (V(G), E(G))$ is a set of vertices such that every vertex in G is adjacent to a vertex in S. In other words $\forall x \in V(G)\ \exists s \in S$: $xs \in E(G)$.

The minimum size of a total dominating set, $\gamma_t(G)$, in a graph, G, is well studied. We will talk about the following new bounds, where $\delta(G)$ is the minimum degree of G and $\Delta(G)$ is the maximum degree in G:

- $\gamma_t(G) \leq \frac{3}{7}|V(G)|$, when $\delta(G) \geq 4$.
- $\gamma_t(G) \leq |V(G)| - \frac{2|E(G)|}{\Delta(G)+2\sqrt{\Delta(G)}}$, when $\Delta(G) \geq 4$.

In fact we can improve the first bound above, if we exclude one specific graph, G_{14}, on 14 vertices. In this case we can obtain the following bound.

- $\gamma_t(G) \leq (\frac{3}{7} - \frac{1}{5943})|V(G)|$, when $\delta(G) \geq 4$ and $G \not\cong G_{14}$.

We will also mention related results and open problems. All the results mentioned in this talk have been obtained by observing that a total dominating set in a graph G is also a transversal in the hypergraph $H(G)$ on the same vertex-set as G and with edge-set $\{N(x)|x \in V(G)\}$. This allows us to use hypergraph techniques in order to obtain the above results.

The number of cycles in 2-factors of line graphs

K. Yoshimoto

MSC2000: 05C38

Let G be a graph with minimum degree at least three. Then it is well known that the line graph $L(G)$ has a 2-factor and $L^2(G)$ is hamiltonian. In this talk, we explain the upper bound of the number of cycles in a 2-factor of $L(G)$, which is best possible. Moreover, we consider the gap between claw-free graphs and line graphs for the properties comparing results on claw-free graphs and a conjecture by Fujisawa et al.

On very sparse circulant (0,1) matrices

N. Zagaglia Salvi

MSC2000: 05C50, 05C10, 15A15

An $n \times n$ matrix A is called generalized i-circulant when it is partitioned into i-circulant submatrices of type $n' \times n$, where $(n, i) = k$ and $n = kn'$. We study generalized i-circulant permutation matrices. Using properties of these matrices we are able to prove that any circulant (0,1)-matrix with three ones per row A is permutation similar to either a particular circulant matrix or to a particular block matrix. As a consequence we determine a lower bound for the permanent of these matrices. Moreover we prove that the bipartite graph associated with A in the usual way has genus 1, but in one case when has genus 0.

Quasi-locally P*(ω) graphs

S. Zenia

(joint work with H . Ait Haddadne)

MSC2000: 05C15, 05C17, 05C69, 05C85

In this paper, we define a new class of graphs called quasi-locally P*(ω) where we give a colouring theorem and propose a polynomial combinatorial algorithm for colouring in polynomial time any perfect graph of this class, for fixed ω.

We will describe a polynomial algorithm for recognizing any graph of this class. We prove that this class contains strictly some classes of graphs.

References.

[AIT 96] Ait haddadne H., Gravier S., On weakly diamond-free Berge graphs. Discrete Mathematics, 159 (1996) 237-240.

[AIT 97] Ait haddadne H., Maffray F., Coloring perfect degenerate graphs, Discrete Mathematics. 163 (1997) 211-215.

[GRA 99] Gravier S., On Tucker vertices of graphs. Discrete Mathematics, 203 (1999)121-131.

[GRO 84] Grotschel M., Lovasz L., Schrijver S., Polynomial algorithm for perfect graphs; Topics one perfect graphs, Annals of Discrete Mathematics 21, 325-356, 1984.

[TUC 87] Tucker C., Coloring perfect K4-e - free graphs, Journal of Combinatorial Theory, B 42 (1987) 313-318.

[TUK 87] Tucker C., A reduction procedure for coloring perfect K4-free graphs, Journal of Combinatorial Theory, B 43 (2) (1987) 151-172.

[ZEN 03] Zenia S., Sur une mthode de coloration de graphes parfaits, Thse de magister, USTHB 2003.

Hypercubes are distance graphs

J. Žerovnik

(joint work with M. Gorše Pihler)

MSC2000: 05C75, 05C12

The ϕ-distance between G_1 and G_2 is

$$d_\phi(G_1, G_2) = \sum |d_{G_1}(u, v) - d_{G_2}(\phi u, \phi v)|,$$

where the sum is taken over all $\binom{n}{2}$ unordered pairs u, v of vertices of G_1. Of course, if $d_\phi(G_1, G_2) = 0$ then ϕ is an isomorphism and $G_1 \cong G_2$, while if $G_1 \not\cong G_2$, then $d_\phi(G_1, G_2) > 0$ for every one-to-one mapping ϕ. This suggests defining the distance $d(G_1, G_2)$ between G_1 and G_2 by

$$d(G_1, G_2) = \min\{d_\phi(G_1, G_2)\},$$

where the minimum is taken over all one-to-one mappings ϕ from $V(G_1)$ to $V(G_2)$. Thus, $d(G_1, G_2) = 0$ if and only if $G_1 \cong G_2$. Hence $d(G_1, G_2)$ can be interpreted as a measure of the similarity of G_1 and G_2, because the smaller the value of $d(G_1, G_2)$, the more similar the structure of G_1 is to that of G_2.

It has been recently conjectured [1] that: **A graph G is a distance graph if and only if G is bipartite** and proved that: every distance graph is bipartite, every even cycle is a distance graph, every tree is a distance graph, the graph $K_{2,n}$ is a distance graph for every positive integer n, etc.

Here we support the conjecture by proving that

Theorem: Every induced subgraph of a hypercube is a distance graph.

Reference.

[1] G.Chartrand, G. Kubicki and M. Schultz, Graph similarity and distance in graphs, Aequationes Math. **55** (1998) 129-145.

Retract-rigid strong graph bundles

B. Zmazek

(joint work with J. Žerovnik)

MSC2000: 05C60, 05C75

Graph bundles generalize the notion of covering graphs and graph products. Let B and F be connected graphs and let $B \boxtimes_{\varphi} F$ be the strong graph bundle over base B with fibre F. A subgraph R of a graph G is a *retract* of G is there an edge-preserving map (retraction) $r : V(G) \to V(R)$ with $r(x) = x$, for all $x \in V(R)$. A graph is retract-rigid if it has no proper retraction.

We show that

(1) if B and F are retract-rigid triangle-free graphs, $G \boxtimes_{\varphi} F$ is also retract-rigid triangle-free graph and

(2) every retract R of $G \boxtimes_{\varphi} F$ is of the form $R = B' \boxtimes_{\varphi} F'$, where B' and F' are isometric subgraphs of B and F, respectively.

(3) For triangle-free base and fibre graphs B and F both B' and F' are retracts of B and F.

(4) There exist retract-rigid graph bundles with base and fibre graphs B and F which admit proper retractions.

A generalised upper bound for the k-tuple domination number

V.E. Zverovich

(joint work with A. Gagarin)

MSC2000: 05C69

We generalise an upper bound for the triple domination number given in [D. Rautenbach and L. Volkmann, New bounds on the k-domination number and the k-tuple domination number. *Discrete Math.* (submitted)]. More precisely, we prove that if G is a graph with $3 \leq k \leq \delta + 1$, then

$$\gamma_{\times k}(G) \leq \frac{\ln(\delta - k + 2) + \ln\left((k-2)d + \sum_{m=2}^{k-1}(2k - 2m - 1)\widehat{d}_m\right) + 1}{\delta - k + 2} n,$$

where $\gamma_{\times k}(G)$ is the k-tuple domination number, δ is the minimal degree, d is the average degree and $\widehat{d}_m$ is the m-degree of G.

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