# Edge-transitive maps and discrete group actions

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- Širáň, Watkins & Tucker (2001) proved that the families are pairwise different,
- Orbanić, Pisanski, Pellicer & Tucker (2011) described all these families in terms of quotient maps and voltage assignments on possibly non-orientable surfaces with non-empty boundary. Their classification of ET's ranges for $\chi \geq -2$.

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Such ET's (on an orientable surface) can be constructed

- Search for suitable normal subgroups of indices $720 = 2^4.3^2.5$, $1440 = 2^5.3^2.5$, or $2880 = 2^6.3^2.5$ in

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- Reconstruct derived maps.

How easy…

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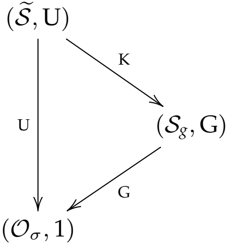
- **3** of them are regular hypermaps (see other M. Conder's list), all are reflexible, two are polytopal, one is polyhedral;
- the remaining **6** maps $\mathrm{Aut}^+ \mathbf{M}$ has two orbits on edges, three of them are polytopal, three of them are not simple, all are reflexible.

# What is behind

- regular branched coverings;
- Riemann uniformization;
- Koebe Theorem.



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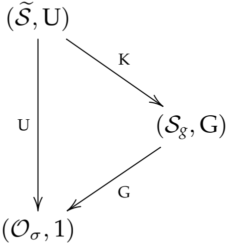


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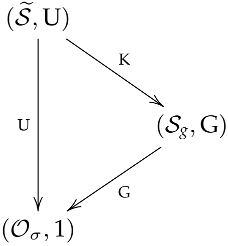


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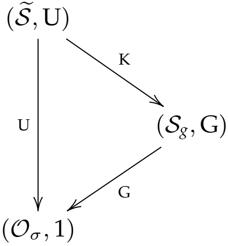


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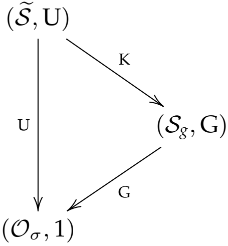


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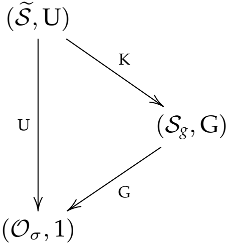


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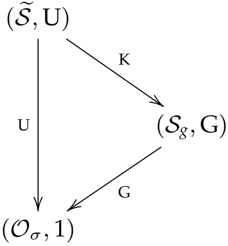


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- action of $\mathrm{G}$ on $\mathcal{S}$ is not fixed-point-free, $\mathcal{O}_\sigma$ is an orbifold;
- $\sigma = (\gamma; \{m_1, m_2, \ldots, m_r\})$ is the signature of the orbifold $\mathcal{O}_\sigma$.

# What is behind

- $p\colon \mathcal{S}_g \to \mathcal{O}_\sigma$ is a branched covering, $\sigma$ follows from Riemann-Hurwitz equation

$$2 - 2g = |\mathrm{G}| \left( 2 - 2\gamma - \sum_{i=1}^{r} \left( 1 - \frac{1}{m_i} \right) \right), \ \forall i : \ m_i \geq 2, \ m_i \mid |\mathrm{G}|;$$

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- $\mathrm{U} \cong \pi_1(\mathcal{O}_\sigma)$ is a Fuchsian group with signature $\sigma$ and the presentation

$$\langle x_1, \ldots, x_r, a_1, b_1, \ldots, a_\gamma, b_\gamma \mid x_1^{m_1} = \ldots = x_r^{m_r} = 1, \prod_{i=1}^{\gamma} [a_i, b_i] \prod_{j=1}^{r} x_j = 1 \rangle;$$

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- $\mathrm{G} = \mathrm{U}/\mathrm{K}$ is finite group acting discretely on $\mathcal{S}_g$, $\mathrm{K} \trianglelefteq \mathrm{U}$ is of finite index and torsion-free.

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- $\bar{\mathbf{M}} = \mathbf{M}/\mathrm{G}$ is the map on orbifold;
- $r$ is regular covering transformation;
- $\mathbf{M}$ can be reconstructed as a derived map with $CT(r) = \mathrm{G} = \mathrm{Aut}^+ \mathbf{M}$;

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- the centre of any face of $\mathbf{M}$ might be a branch point

# Isomorphisms

Let $\mathbf{M}$ and $\mathbf{N}$ be maps on orbifolds. The mapping $\varphi\colon \mathbf{M} \to \mathbf{N}$ is isomorphism of maps on orbifolds if

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- $\varphi$ maps branch points to branch points;
- $\varphi$ preserves branch indices.

# Quotients of edge transitive maps

- $\bar{\mathbf{M}} = \mathbf{M}/\operatorname{Aut}^+\mathbf{M}$ is sitting on an orbifold;
- $\bar{\mathbf{M}}$ has at most two edges;
- if $\bar{\mathbf{M}}$ has two edges, then a reflection of $\bar{\mathbf{M}}$ transposes them.

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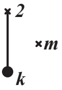
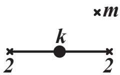
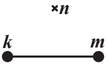


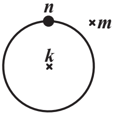
E1



E2



E3



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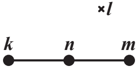
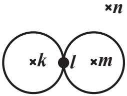
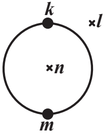


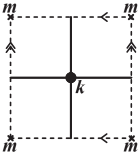
E4



E4*



E5



E6

# How derive maps upstairs

- $T$-reduced voltage assignment on $\bar{\mathbf{M}}$ is $\xi : V \cup D \to G$ such that
  1. all darts $D^+(T)$ on the rooted spanning tree $(T, x_0)$ receive trivial voltages,
  2. $\xi_{xL} = \xi_x^{-1}$ for all $x \in D$,
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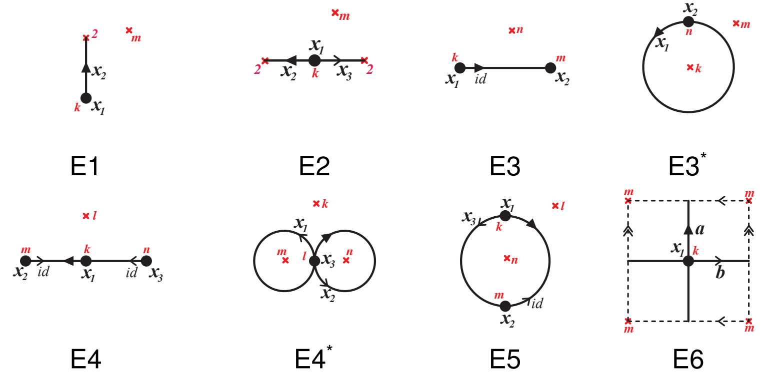
  3. $G = \langle \{\xi_x : x \in D \cup V\} \rangle$.
- Derived map $\mathbf{M} = \bar{\mathbf{M}}^\xi = (D^\xi, R^\xi, L^\xi)$, $D^\xi = D \times \mathrm{G}$ is given by

$$(x,g)R^\xi = \begin{cases} (xR, g \cdot \xi_v), & x \in D^+(T), \\ (xR, g), & \text{otherwise} \end{cases}$$

$$(x,g)L^\xi = (xL, g \cdot \xi_x)$$

# Maps with voltages



# Computation

- given $g > 1$ take all (numerical) solutions of RH equation – tuples $|\mathrm{G}|, \sigma$;
- search for normal subgroups of index $d.|\mathrm{G}|$;
- check whether $|\mathrm{G}| = |\operatorname{Aut}^+ \mathbf{M}|$.

| | | |
|---|---|---|
| E1 | $\langle x_1, x_2 \mid x_1^k = x_2^2 = (x_1^{-1}x_2)^m = 1\rangle$ | $d = 1$ |
| E2 | $\langle y_1, y_2, y_3, r \mid y_1^k = y_2^2 = y_3^2 = (y_1^{-1}y_2y_3)^m = 1,$ $r^2 = 1, y_1^r = y_1^{-1}, y_2^r = y_3, y_3^r = y_2\rangle$ | $d = 2$ |
| E3 | $\langle x_1, x_2 \mid x_1^k = x_2^l = (x_2^{-1}x_1^{-1})^m = 1\rangle$ | $d = 1$ |
| E4 | $\langle y_1, y_2, y_3, r \mid y_1^k = y_2^l = y_3^m = (y_1^{-1}y_2^{-1}y_3^{-1})^n = 1, \rangle$ $r^2 = 1, y_1^r = y_2^{-1}, y_2^r = y_1^{-1}, y_3^r = y_3^{-1}\rangle$ | $d = 2$ |
| E5a | $\langle y_1, y_2, y_3, r \mid y_1^k = y_2^l = (y_3y_2^{-1})^m = (y_3^{-1}y_1^{-1})^n = 1, \rangle$ $r^2 = 1, y_1^r = y_1^{-1}, y_2^r = y_2^{-1}, y_3^r = (y_2y_3y_2^{-1})^{-1}\rangle$ | $d = 2$ |
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| E6a | $\langle z, a, b, s \mid z^k = (z^{-1}ab^{-1}a^{-1}b)^m = 1,$ $s^2 = 1, z^s = z^{-1}, a^s = b^{-1}, b^s = a^{-1}\rangle$ | $d = 2$ |
| E6b | $\langle z, a, b, s \mid z^k = (z^{-1}ab^{-1}a^{-1}b)^m = 1,$ $s^2 = 1, z^s = z^{-1}, a^s = b, b^s = a\rangle$ | $d = 2$ |

# Results

| $g$ | Family / Subfamily | | | | | | | | Maps |
|---|---|---|---|---|---|---|---|---|---|
| | E1 | E2 | E3 | E4 | E5a | E5b | E6a | E6b | |
| 2 | 10 | 1 | 18 | 2 | 44 | 0 | 1 | 0 | 76 |
| 3 | 20 | 2 | 46 | 6 | 108 | 0 | 1 | 1 | 184 |
| 4 | 20 | 7 | 53 | 13 | 137 | 0 | 6 | 2 | 238 |
| 5 | 26 | 11 | 54 | 20 | 177 | 0 | 5 | 4 | 297 |
| 6 | 23 | 9 | 70 | 16 | 221 | 2 | 7 | 4 | 352 |
| 7 | 27 | 19 | 80 | 38 | 317 | 0 | 10 | 8 | 499 |
| 8 | 24 | 9 | 68 | 18 | 237 | 3 | 8 | 6 | 373 |
| 9 | 52 | 39 | 141 | 77 | 567 | 0 | 26 | 16 | 918 |
| 10 | 54 | 26 | 158 | 56 | 544 | 0 | 27 | 16 | 881 |

- maps of different families cannot be isomorphic;

# Results

| $g$ | Family / Subfamily | | | | | | | | Maps |
|---|---|---|---|---|---|---|---|---|---|
| | E1 | E2 | E3 | E4 | E5a | E5b | E6a | E6b | |
| 2 | 10 | 1 | 18 | 2 | 44 | 0 | 1 | 0 | 76 |
| 3 | 20 | 2 | 46 | 6 | 108 | 0 | 1 | 1 | 184 |
| 4 | 20 | 7 | 53 | 13 | 137 | 0 | 6 | 2 | 238 |
| 5 | 26 | 11 | 54 | 20 | 177 | 0 | 5 | 4 | 297 |
| 6 | 23 | 9 | 70 | 16 | 221 | 2 | 7 | 4 | 352 |
| 7 | 27 | 19 | 80 | 38 | 317 | 0 | 10 | 8 | 499 |
| 8 | 24 | 9 | 68 | 18 | 237 | 3 | 8 | 6 | 373 |
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|---|---|---|---|---|---|---|---|---|---|
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| 2 | 10 | 1 | 18 | 2 | 44 | 0 | 1 | 0 | 76 |
| 3 | 20 | 2 | 46 | 6 | 108 | 0 | 1 | 1 | 184 |
| 4 | 20 | 7 | 53 | 13 | 137 | 0 | 6 | 2 | 238 |
| 5 | 26 | 11 | 54 | 20 | 177 | 0 | 5 | 4 | 297 |
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- see `http://www.savbb.sk/~karabas/science.html#etran`;
- all maps as $(D; R, L)$, we can derive those with $(D; RL, L)$;
- construction of non-orientable maps needs more effort.