MST209
DIAGNOSTIC QUIZ

Am I ready to start on MST209?

The diagnostic quiz below is designed to help you to answer this question. This document also contains some advice on preparatory work that you may find useful before starting on MST209 (see below and pages 8–9).

The mathematical skills required for MST209 can be separated into two levels:

A those that are assumed but not discussed at all in the course;

B those that are reviewed in the course.

The diagnostic quiz below is divided into corresponding sections A and B.

To be ready to start on MST209, you should be confident about level A topics. You should also have met level B topics before, and be able to handle them with the brief reminder provided in *Unit 1* of the course.

MST209 uses the computer algebra package Mathcad 2001i. The Mathcad software will be sent to you in January along with the *Computing Booklet*, which introduces the software. If Mathcad is completely new to you, then you should allow one week of study prior to the course to work on the *Computing Booklet*. (If you have met any version of Mathcad before, then your study of this should take much less time.)

Try the questions now, and then see the notes on pages 8–9 of this booklet to see if you are ready for MST209.

Diagnostic Quiz – Questions

LEVEL A

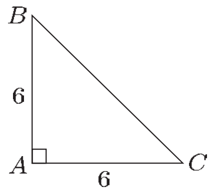
1 Using a calculator, give the values of each of the following to three decimal places:

(a) $\tan(1.2)$ (where 1.2 is in radians);

(b) $e^{-2.731}$;

(c) $\ln(4/27)$.

2 In the triangle BAC, the angle BAC is a right angle, and the sides AB and AC are each of length 6.

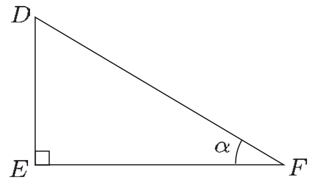


(a) Give each of the angles in triangle ABC in degrees and in radians.

(b) What is the length BC?

(c) What is the area of triangle ABC?

3 (a) In the triangle DEF, the angle DEF is a right angle, and angle EFD is α. Write down each of $\cos\alpha$, $\sin\alpha$ and $\tan\alpha$ as ratios of sides in the triangle DEF.



(b) Give the values of $\cos(180°)$ and $\sin(270°)$.

4 Solve for x each of the following equations.

(a) $3x + 4 = 10$

(b) $3(x + 3) - 7(x - 1) = 0$

(c) $\dfrac{2}{1 + x} = \dfrac{3}{2 - x}$

(d) $\sqrt{x^2 + 7} = 4$

5 (a) Make t the subject of the equation

$$x = x_0 - \tfrac{1}{2}gt^2.$$

(b) Make x the subject of the equation

$$\sqrt{\frac{x - 2}{x + 3}} = t.$$

6 Give the equation of the straight line passing through the points $y = 2$ when $x = 0$ and $y = 8$ when $x = 2$. What is the gradient of this line?

7 If $y(x) = 3 + 2x - \sin(2x)$, what is $y\left(\frac{\pi}{2}\right)$?

LEVEL B

8 (a) Solve for y the equation

$$2y^2 - 4y + 1 = 0.$$

(b) Solve for λ the equation

$$\lambda^2 + 4\lambda + 4 = 0.$$

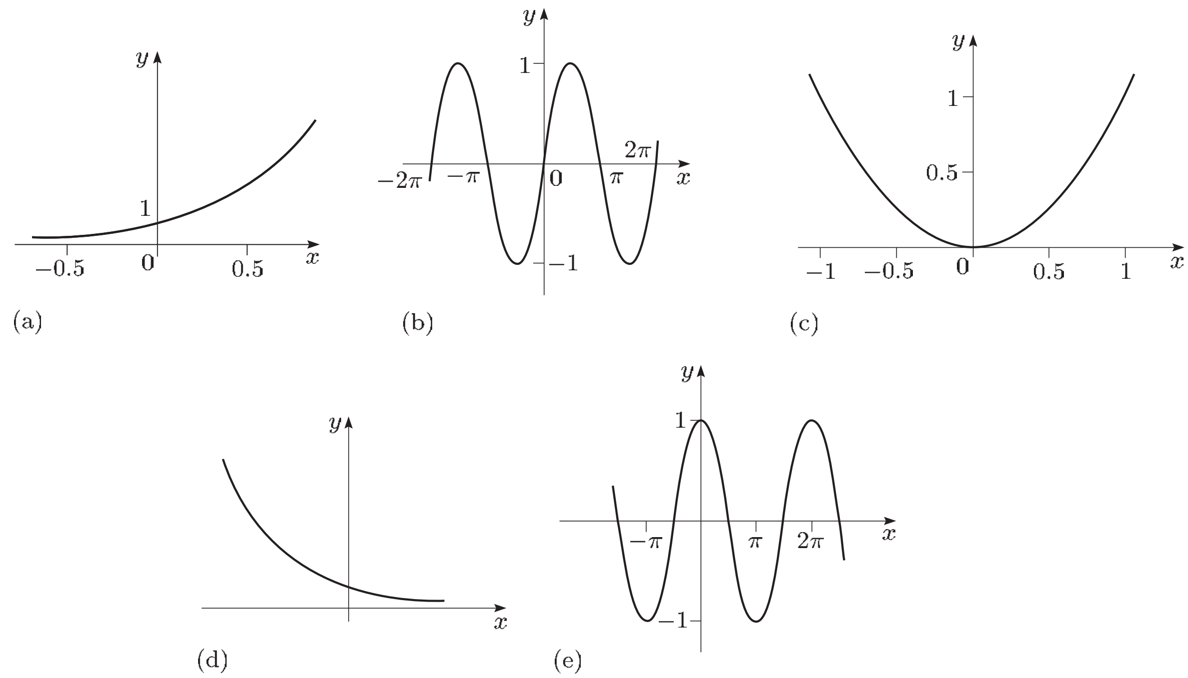
9 Solve the following simultaneous equations for x and y:

$$2x - y = 3,$$
$$3x + y = 2.$$

10 You plan to make some cakes and some biscuits for a charity sale. Each cake uses 2 eggs and 0.11 kg of butter. Each batch of biscuits requires 1 egg and 0.15 kg of butter. You have only 12 eggs and 1 kg of butter, and you want to use all of these (so far as is possible). (Each recipe also requires other ingredients, but you have plenty of those.) How many cakes and how many batches of biscuits should you make?

11 Five graphs are given in parts (a)–(e) of the figure below. Each graph is that of one of the functions (i)–(v). Match each graph with the appropriate function.

Functions: (i) $y(x) = e^{-x}$, (ii) $y(x) = e^{2x}$, (iii) $y(x) = \sin x$, (iv) $y(x) = \cos x$, (v) $y(x) = x^2$.



12 Simplify each of the following.

(a) x^2x^5 (b) x^3/x^4 (c) $(x^2)^3$ (d) $9^{1/2}$

13 Express $(e^{-2x} \times e^{3x})^2$ in the form e^y.

14 Express $\frac{1}{2}\ln(25) + 3\ln(\frac{1}{2})$ in the form $\ln(y)$.

15 Show that $y = \ln(2e^{-x/2})$ is the equation of a straight line. What is the gradient of this line?

16 Solve for y the equation

$$\ln(y) = 2\ln(x) - 1.$$

17 If $|x - 2| < 10^{-2}$, what range of values can x take?

18 Solve for ν the equation below (where $m \neq 0$):

$$-\frac{m}{gr} = -\frac{\mu m}{\nu^2}.$$

19 (a) What solutions for x has the equation $\sin x = 1$?

(b) What value does your calculator give for $\arcsin(1)$?

20 What is $\cos^2 \alpha + \sin^2 \alpha$ (where α may be any real number)?

21 Use the trigonometric identity $\cos(a + b) = \cos a \cos b - \sin a \sin b$, and particular values of a and b, to simplify $\cos(\pi + x)$.

22 Find the local maxima and minima of the function

$$y(x) = 2x^3 - 3x^2 - 12x + 6.$$

23 (a) Find $\dfrac{ds}{dt}$ where $s = 5e^{3t}$.

(b) Find $y'(t)$ where $y(t) = 3t^5 - 10\sqrt{t}$.

(c) Find $\dfrac{dz}{dx}$ where $z = 14\sin(x/8)$.

24 Evaluate each of the following integrals.

(a) $\displaystyle\int (1 + 6x^3)\,dx$ (b) $\displaystyle\int_0^{\pi} \sin(3t)\,dt$

25 Suppose that

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2},$$

where m, r, g and μ are positive.

(a) Rearrange this inequality by multiplying each side first by $-\nu^2$, then by gr/m.

(b) In terms of the other parameters, what is the largest value that ν can take?

26 (a) (i) Find $y'(t)$ where $y(t) = t\sin(3t)$.

(ii) Find $\dfrac{dx}{dt}$ where $x = \ln(t^3 + 1)$.

(b) Find the velocity at time $t = 3$ of an object whose position at time t is given by $x(t) = e^{-2t}\cos(\frac{\pi}{3}t)$.

27 (a) Use integration by substitution to find $\displaystyle\int x^2 \exp(2 + 3x^3)\,dx$.

(b) Use integration by parts to find $\displaystyle\int x \ln x\,dx$.

28 Express the complex number $(1 + i)(3 + 2i)$ in the form $a + bi$.

29 Find the imaginary part of the complex number $(e^{2+i\pi})^3$.

Diagnostic Quiz – Answers

Where a topic is covered in *Unit 1* of MST209, we indicate the appropriate section or subsection of the unit at the end of the answer.

LEVEL A

1 (a) 2.572

(b) 0.065

(c) −1.910

2 (a) $\angle ABC = \angle ACB = 45° = \frac{\pi}{4}$ radians.
$\angle BAC = 90° = \frac{\pi}{2}$ radians.

(b) By Pythagoras's Theorem,

$$BC^2 = AB^2 + AC^2 = 6^2 + 6^2 = 72,$$

so

$$BC = \sqrt{72} = 6\sqrt{2}.$$

(c) The area of a triangle equals half its base times its height, so the area of triangle ABC is

$\frac{1}{2} \times 6 \times 6 = 18$ square units.

3 (a) $\cos\alpha = \dfrac{EF}{DF}$, $\sin\alpha = \dfrac{DE}{DF}$, $\tan\alpha = \dfrac{DE}{EF}$.

(b) $\cos(180°) = -1$, $\sin(270°) = -1$.

4 (a) $3x + 4 = 10$
$3x = 6$
$x = 2$

(b) $3(x+3) - 7(x-1) = 0$
$3x + 9 - 7x + 7 = 0$
$-4x + 16 = 0$
$x = 4$

(c) $\dfrac{2}{1+x} = \dfrac{3}{2-x}$
$2(2-x) = 3(1+x)$
$4 - 2x = 3 + 3x$
$5x = 1$
$x = \frac{1}{5}$

(d) $\sqrt{x^2+7} = 4$
$x^2 + 7 = 4^2 = 16$
$x^2 = 9$
$x = \pm\sqrt{9}$
So $x = 3$ or $x = -3$.

5 (a) $x = x_0 - \frac{1}{2}gt^2$
$gt^2 = 2(x_0 - x)$
$t = \pm\sqrt{\dfrac{2}{g}(x_0 - x)}$

(b) $\sqrt{\dfrac{x-2}{x+3}} = t$
$\dfrac{x-2}{x+3} = t^2$
$x - 2 = t^2(x+3) = t^2x + 3t^2$
$x(1-t^2) = 2 + 3t^2$
$x = \dfrac{2+3t^2}{1-t^2}$

6 The equation of a straight line has the form

$y = mx + c.$

To satisfy the given conditions, the constants m and c must satisfy the equations

$2 = c$ (since $y = 2$ when $x = 0$),
$8 = 2m + c$ (since $y = 8$ when $x = 2$).

Thus $c = 2$ and $m = 3$, so the required equation is

$y = 3x + 2.$

The gradient of this line is given by m, and so is 3.

7
$$\begin{aligned} y\left(\tfrac{\pi}{2}\right) &= 3 + 2 \times \tfrac{\pi}{2} - \sin\left(2 \times \tfrac{\pi}{2}\right) \\ &= 3 + \pi - \sin\pi \\ &= 3 + \pi - 0 \\ &= 3 + \pi \end{aligned}$$

LEVEL B

8 Use the formula for solving a quadratic equation.

(a) $y = \dfrac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$
$= \dfrac{4 \pm \sqrt{8}}{4} = 1 \pm \frac{1}{2}\sqrt{2}$

(b) $\lambda = \dfrac{-4 \pm \sqrt{16-16}}{2} = -2$

(The expression $\lambda^2 + 4\lambda + 4$ is a perfect square, $(\lambda+2)^2$.)

[*Unit 1*, Subsection 2.3]

9 Adding the equations gives $5x = 5$, so $x = 1$. Then the first equation gives $2 - y = 3$, so $y = -1$.

The solution is $x = 1$, $y = -1$.

[*Unit 1*, Subsection 2.2]

10 Suppose that you make C cakes and B batches of biscuits. To use all the ingredients, you would need

$2C + B = 12$ (for the eggs),
$C(0.11) + B(0.15) = 1$ (for the butter).

The first equation gives $B = 12 - 2C$, and then the second equation gives

$C(0.11) + (12 - 2C)(0.15) = 1.$

Solving this equation for C gives $C = 4.2$ (to one decimal place), and then $B = 3.6$.

You might be able to make the equivalent of 4.2 cakes (by making 4 cakes each just a little bit larger than the recipe). But this would imply using 8.4 eggs on the cakes, which does not look very feasible (though you could try to divide an egg).

The best bet would seem to be to make 4 cakes and 4 batches of biscuits. This will use up all 12 eggs. It really needs 1.04 kg of butter – a bit more than you have. Either steal a scrap from your butter dish, or, if that's not possible, skimp a bit on the butter for the biscuits. (Scale down the other ingredients very slightly if you're worried, so that you're being slightly generous on the eggs rather than mean on the butter.)

The main point here is that, when you are applying mathematics, you may need to use your common sense to relate the results of a mathematical calculation to the real problem.

11 The matching is as follows.

(a)(ii) (b)(iii) (c)(v) (d)(i) (e)(iv)

[*Unit 1*, Sections 2 and 3]

12 We use the rules for manipulating indices.

(a) $x^2x^5 = x^{2+5} = x^7$

(b) $x^3/x^4 = x^{3-4} = x^{-1} (= 1/x)$

(c) $(x^2)^3 = x^{2\times 3} = x^6$

(d) $9^{1/2} = \sqrt{9} = 3$

[*Unit 1*, Subsection 2.4]

13 $(e^{-2x} \times e^{3x})^2 = (e^{3x-2x})^2 = (e^x)^2 = e^{2x}$

[*Unit 1*, Subsection 2.4]

14
$$\begin{aligned}\tfrac{1}{2}\ln(25) + 3\ln(\tfrac{1}{2}) &= \ln(\sqrt{25}\,) + \ln((\tfrac{1}{2})^3) \\ &= \ln(5) + \ln(\tfrac{1}{8}) \\ &= \ln(\tfrac{5}{8})\end{aligned}$$

[*Unit 1*, Subsection 2.4]

15 Using the properties of exponentials and logarithms,

$$\ln(2e^{-x/2}) = \ln(2) + \ln(e^{-x/2}) = \ln(2) - \tfrac{1}{2}x,$$

so

$$y = \ln(2) - \tfrac{1}{2}x.$$

This is the equation of a straight line.

The gradient of the straight line is $-\frac{1}{2}$ (the coefficient of x).

[*Unit 1*, Subsections 2.2 and 2.4]

16 If $\ln(y) = 2\ln(x) - 1$, then taking exponentials of each side gives

$$\begin{aligned}\exp(\ln(y)) &= \exp(2\ln(x) - 1) \\ y &= \exp(\ln(x^2) - 1) \\ &= \exp(\ln(x^2))/\exp(1) \\ &= x^2/e.\end{aligned}$$

[*Unit 1*, Subsection 2.4]

17 Recall that $|y|$ means y if $y \geq 0$, and $-y$ if $y < 0$. If $|x - 2| < 10^{-2}$, then $-10^{-2} < x - 2 < 10^{-2}$, so $2 - 10^{-2} < x < 2 + 10^{-2}$, i.e. $1.99 < x < 2.01$.

[*Unit 1*, Subsection 1.1]

18 If $-\dfrac{m}{gr} = -\dfrac{\mu m}{\nu^2}$, then multiplying each side by $-\dfrac{\nu^2}{m}$ gives

$$\left(-\frac{\nu^2}{m}\right)\left(-\frac{m}{gr}\right) = \left(-\frac{\nu^2}{m}\right)\left(-\frac{\mu m}{\nu^2}\right),$$

i.e. $\dfrac{\nu^2}{gr} = \mu$. Hence $\nu = \pm\sqrt{\mu g r}$.

19 (a) $\sin x = 1$ when $x = \frac{\pi}{2}$, or when x differs from $\frac{\pi}{2}$ by a multiple of 2π.

(b) My calculator gives $\arcsin(1) = 90$, but that is because it is working in degrees. If your calculator is working in radians (as it will need to be for MST209), then it should give

$$\arcsin(1) = 1.570796327 \quad (\text{i.e. } \tfrac{\pi}{2}).$$

[*Unit 1*, Subsections 3.1 and 3.2]

20 $\cos^2\alpha + \sin^2\alpha = 1$.

[*Unit 1*, Subsection 3.3]

21 We have

$$\begin{aligned}\cos(\pi + x) &= \cos\pi\cos x - \sin\pi\sin x \\ &= (-1)\cos x - (0)\sin x \\ &= -\cos x.\end{aligned}$$

[*Unit 1*, Subsection 3.3]

22 To find local maxima and minima, first find the stationary points, where $\dfrac{dy}{dx} = 0$.

Differentiating $y = 2x^3 - 3x^2 - 12x + 6$ gives

$$\frac{dy}{dx} = 6x^2 - 6x - 12.$$

So to find the stationary points, solve

$$6x^2 - 6x - 12 = 0,$$

i.e.

$$x^2 - x - 2 = 0.$$

To solve this quadratic equation, you can either use the formula, or factorize to obtain

$$(x - 2)(x + 1) = 0.$$

Thus there are stationary points at $x = 2$ and $x = -1$.

Now $\dfrac{d^2y}{dx^2} = 12x - 6$.

At $x = -1$, this is negative, so there is a local maximum at $x = -1$, of value $y = 13$.

At $x = 2$, this second derivative is positive, so there is a local minimum at $x = 2$, of value $y = -14$.

[*Unit 1*, Subsection 5.1]

23 (a) If $s = 5e^{3t}$, then

$$\frac{ds}{dt} = 5(3e^{3t}) = 15e^{3t}.$$

(b) If $y(t) = 3t^5 - 10\sqrt{t}$, then

$$y'(t) = 15t^4 - 5t^{-1/2}.$$

(c) If $z = 14\sin(x/8)$, then

$$\frac{dz}{dx} = \tfrac{14}{8}\cos(x/8) = \tfrac{7}{4}\cos(x/8).$$

[*Unit 1*, Subsection 5.1]

24 (a) This is an indefinite integral:

$$\begin{aligned}\int (1 + 6x^3)\,dx &= x + \tfrac{6}{4}x^4 + c\\ &= x + \tfrac{3}{2}x^4 + c,\end{aligned}$$

where c is an arbitrary constant.

(b) This is a definite integral:

$$\begin{aligned}\int_0^{\pi} \sin(3t)\,dt &= \left[-\tfrac{1}{3}\cos(3t)\right]_0^{\pi}\\ &= -\tfrac{1}{3}(\cos(3\pi) - \cos(0))\\ &= -\tfrac{1}{3}(-1 - 1) = \tfrac{2}{3}.\end{aligned}$$

[*Unit 1*, Subsection 6.2]

25 We have

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2}.$$

(a) The quantity $-\nu^2$ is negative, so on multiplying both sides by $-\nu^2$, we must reverse the inequality:

$$\frac{m}{gr}\nu^2 \leq \mu m.$$

Then (since gr/m is positive)

$$\nu^2 \leq \mu g r.$$

(b) The largest value that ν can take is $\sqrt{\mu g r}$.

26 (a) (i) To differentiate $y(t) = t\sin(3t)$, use the product rule. We obtain

$$y'(t) = \sin(3t) + 3t\cos(3t).$$

(ii) To differentiate $x = \ln(t^3 + 1)$, use the composite (or 'function of a function') rule. We obtain

$$\frac{dx}{dt} = 3t^2 \times \frac{1}{t^3+1} = \frac{3t^2}{t^3+1}.$$

(b) To find the velocity $v(t)$ of the object, we differentiate the expression for its position, i.e.

$$\begin{aligned}v(t) &= \frac{dx}{dt}\\ &= -2e^{-2t}\cos(\tfrac{\pi}{3}t) - \tfrac{\pi}{3}e^{-2t}\sin(\tfrac{\pi}{3}t)\\ &= -e^{-2t}(2\cos(\tfrac{\pi}{3}t) + \tfrac{\pi}{3}\sin(\tfrac{\pi}{3}t)).\end{aligned}$$

So at time $t = 3$, the object's velocity is

$$\begin{aligned}v(3) &= -e^{-6}(2\cos\pi - \tfrac{\pi}{3}\sin\pi)\\ &= -e^{-6}(2(-1) + \tfrac{\pi}{3}(0))\\ &= 2e^{-6}\\ &\simeq 0.005.\end{aligned}$$

[*Unit 1*, Subsection 5.2]

27 (a) Integration by substitution uses the formula

$$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du.$$

With $u(x) = 2 + 3x^3$, we have $\dfrac{du}{dx} = 9x^2$, and then

$$\begin{aligned}\int x^2\exp(2 + 3x^3)\,dx &= \int \tfrac{1}{9}\frac{du}{dx}\exp u\,dx\\ &= \tfrac{1}{9}\int \exp u\,\frac{du}{dx}\,dx\\ &= \tfrac{1}{9}\int \exp u\,du\\ &= \tfrac{1}{9}\exp u + c\\ &= \tfrac{1}{9}\exp(2 + 3x^3) + c,\end{aligned}$$

where c is an arbitrary constant.

(b) Integration by parts uses the formula

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

With $f(x) = \ln x$ and $g(x) = \tfrac{1}{2}x^2$, we have $f'(x) = 1/x$ and $g'(x) = x$, and then

$$\begin{aligned}\int x\ln x\,dx &= \tfrac{1}{2}x^2\ln x - \int \tfrac{1}{2}x\,dx\\ &= \tfrac{1}{2}x^2\ln x - \tfrac{1}{4}x^2 + c,\end{aligned}$$

where c is an arbitrary constant.

[*Unit 1*, Subsection 6.3]

28 Since $i^2 = -1$, we have

$$\begin{aligned}(1+i)(3+2i) &= 3 + 2i + 3i + 2i^2\\ &= 3 + 5i - 2\\ &= 1 + 5i.\end{aligned}$$

[*Unit 1*, Subsection 4.1]

29 We have

$$\begin{aligned}(e^{2+i\pi})^3 &= e^{(2+i\pi)\times 3}\\ &= e^{6+3i\pi}\\ &= e^6(\cos(3\pi) + i\sin(3\pi)).\end{aligned}$$

The imaginary part of this is

$$e^6\sin(3\pi) = 0.$$

[*Unit 1*, Subsection 4.2]

Materials that you can use to prepare for MST209

MST209 Bridging Material

This is designed for students who have done MST121 (or equivalent), but not MS221. So long as you are familiar with the calculus in MST121 (or its equivalent), then this Bridging Material should take two or three weeks to study. This topics revised, in order of importance to MST209, are:

- techniques for differentiating products, quotients and composite functions;
- integration methods, namely integration by parts and by substitution;
- complex numbers.
- techniques for approximating functions using Taylor polynomials;

MST209 expects you to be proficient at differentiation 'by hand'. The integration methods are needed less often as most of the integrals can be done by using the table of standard integrals given in the course Handbook. Complex numbers and Taylor polynomials are used only a little in MST209. However, you are still expected to understand these ideas.

You can often use Mathcad, the computer algebra package, to help with integration (as well as with the differentiation of complicated functions).

The MST209 Bridging Material is available on the MST209 public website

`http://mcs.open.ac.uk/MST209`

Unit 1 of MST209

This reviews a number of topics used in MST209. This unit does not introduce these topics from scratch; rather, it provides a reminder of them, and an opportunity to refresh your memory and practise techniques. Topics covered include:

- sequences and limits;
- standard functions, such as linear, quadratic, exponential and logarithm functions, and algebraic manipulations involving these;
- trigonometric functions and identities involving these;
- complex numbers;
- differentiation;
- integration;
- use of the computer to manipulate algebraic expressions, draw graphs and solve equations.

This unit includes the topics in the Bridging Material (except Taylor polynomials), but the Bridging Material covers them more thoroughly.

Taylor polynomials are revised later in MST209.

Later units of MST209 expect you to be proficient in the ideas and methods covered in *Unit 1*, particularly the use of the various standard functions, including the trigonometric functions, and manipulation of expressions involving them, differentiation, and integration using the table of standard integrals given in the course Handbook. The other topics in *Unit 1* are used at times later in the course, but are less crucial to your later work.

Computing Booklet

This covers installing and using all of the course software, both the multimedia presentations and Mathcad (the computer algebra package). It contains activities designed to give you the basic skills in working with this software.

What can I do to prepare for MST209?

Use of the following flowchart in connection with the diagnostic quiz should help you to decide whether MST209 is an appropriate course for you, and what you should do by way of preparation before the course starts.

Most or all of level A is completely unfamiliar. →	Request an interview with a Regional Advisor. Consider MU120 or MST121.
Some of levels A and B is familiar, but some is not. →	Consider MST121 (and/or MS221).
I have met nearly all the topics in levels A and B before, but need some practice. →	You will need to spend some time in preparing for MST209. Study the MST209 Bridging Material. Also, start work on *Unit 1* as soon as possible, so that you can spend more than one week on it if need be.
I have met all the topics in the quiz before, but need to practise some of them. →	Start work on *Unit 1* as soon as possible, so that you can spend more than one week on it if need be.
All the mathematics is fine, but I am not sure about the computing. →	Start work on the *Computing Booklet* as soon as possible.
I think that I am already fully prepared to start on MST209. →	Good! It would still be advisable to look at the *Computing Booklet* and *Unit 1* as soon as possible, to check that there is nothing on which you want to spend extra time.